Frame synchronization of OFDM systems in frequency selective fading channels

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Abstract

This paper investigates the topic of frame synchronization for systems based on the OFDM principle. After introducing the system model we discuss the task of frame-synchronization and analyze the effects of a frame offset. From this we derive an appropriate criterion to measure the performance of synchronization algorithms under multipath conditions. We examine two algorithms for frame synchronization: The first algorithm is based on an evaluation of periodic structures. Since it is completely independent of a frequency offset it may be applied for system acquisition and burst synchronization. If the requirements are very high as in terrestrial TV broadcasting a second algorithm is needed to attain sufficient accuracy. The algorithm presented for this purpose is based on the evaluation of the channel estimate, so no further training data is required.

I Introduction

Multi carrier systems such as *orthogonal frequency division multiplexing* (OFDM) are well known to allow a bandwidth-efficient transmission over strongly frequency-selective channels at a moderate implementation effort [1]. A coherent system based on this method has just been selected for the new European digital terrestrial TV standard (DVB-T) [2] and OFDM-based systems are considered for broadband wireless indoor systems [3].

The estimation of the frame start position determines the alignment of the FFT-window in the receiver with the useful portion of the OFDM symbol. A false estimate leads to ISI which may disturb the orthogonality of the system and cause essential degradation due to *inter channel interference* (ICI). In the case of a coherent system in addition the a priori assumptions made for channel estimation may be violated which results in a degradation of the channel estimate. It is useful to analyze these effects to find a proper measure for the performance of synchronization algorithms.

II System Model

We consider an OFDM System having a FFT length of N_{fft} where a total of N_u subcarriers are used for transmission. The transmitted signal $s(t)$ is generated via an inverse FFT operation of the subcarrier symbols $a_{n,l}$ and zeros representing the virtual carriers. To prevent ISI a guard interval of length Δ is placed in front of the useful portion T_u of the signal according to figure 3. If Δ is chose bigger than the maximum delay of the *channel impulse response* (cir) the orthogonality of the system is preserved under multipath conditions. The samples of the transmitted baseband signal are given by

$$s(k) = \frac{1}{\sqrt{N_{fft}}} \sum_{l=-N_u/2}^{(N_u/2)-1} a_{n,l}\, e^{j2\pi \frac{lk}{N_{fft}}} \qquad (1)$$
$$for \quad -N_\Delta \leq k \leq N_{fft}-1$$

where N_Δ are the samples corresponding to the guard interval. After transmission over a multipath channel the samples at the receiver are

$$r(k) = \sum_{i=0}^{N_h} s(k-i)\cdot h_n(i) + n(k) \qquad (2)$$

where $h_n(i)$ is the sampled complex *channel impulse response* (cir) and $n(k)$ is complex white Gaussian noise. After correctly removing the guard interval the signal is demodulated via a FFT yielding the received subcarriere symbols at timeslot n and subcarrier l

$$z_{n,l} = a_{n,l}\ H(n,l) \ +\ n_{n,l} \qquad (3)$$

where $n_{n,l}$ again is complex white Gaussian noise and

$$H(n,l) = \sum_{k=0}^{N_h-1} h_n(k) \quad e^{j\,2\pi \frac{lk}{N}} \qquad (4)$$

is the channel transfer function at frequency $f_l = \frac{l}{T_u}$.

If coherent modulation is used for transmission the influence of the channel $H(n,l)$ has to be estimated and compensated. A well-known approach which may be applied for receivers for the DVB-T system is channel estimation via interpolation i.e. using Wiener filters [4, 5]. Let's assume that known training symbols $p_{n,k}$ are transmitted according to figure 1 where D_f is the distance of the symbols in frequency direction. The pilots provide samples of the *channel transfer function* (ctf), that can be used for estimation via interpolation in time and frequency direction. For our analysis we assume that interpolation

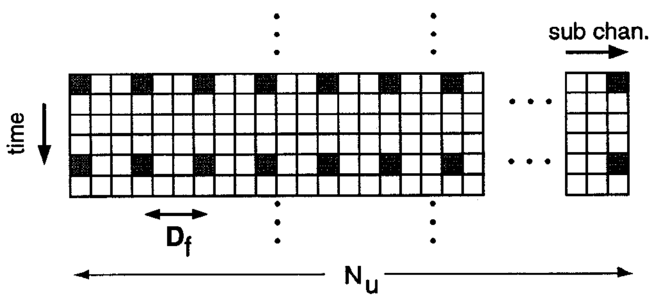

Figure 1: Frame format for channel estimation

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in time provides the samples for OFDM symbols without pilots and focus on interpolation in frequency direction. Let's assume that $N_u = N_{fft}$ and let P be the set of subcarrier indices where the pilots are located. For each OFDM symbol carrying pilots we can define the sequence

$$H_s(n,l) = \begin{cases} D_f \cdot z_{n,l}/p_{n,l} & if \quad l \in P \\ 0 & else \end{cases} \tag{5}$$

Then the interpolation may be expressed as

$$\widetilde{H}(n,l) = \sum_{i=-L/2}^{L/2} H_s(n,l-i) \cdot w(i) \tag{6}$$

where $w(l)$ is a properly designed interpolation filter of length L_h. In the transformation domain the process may be interpreted as depicted in figure 2. An estimate of the cir which is periodic with $\frac{T_u}{D_f}$ is filtered by the interpolation filter with transfer function $W(k)$ which will usually be chosen to be greater than the guard-interval. The quality of the estimate is essential for the system performance. Simulation results indicate that in order to be negligible the MSE of the channel estimate must be about one decade below the variance of the channel noise. This requires that the cir is located completely within the interpolation window.

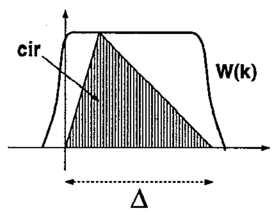

Figure 2: Channel estimation via interpolation

III Effects of frame misalignment

The main task of frame synchronization is depicted in figure 3. Due to the multipath propagation a part of the guard interval will be affected by reflections from the preceding symbol. The frame synchronization must provide a portion oft the samples $r(k)$ of length N_{fft} that is influenced by one transmitted symbol only. Let's define the positions $k = n(N_{fft} + N_\Delta)$ as the correct position of the FFT window and k_ϵ the offset w.r.t. to this position. In the case of a misalignment we can identify two situations depicted in figure 3.

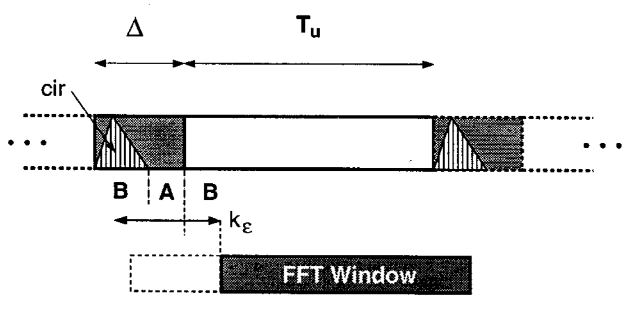

Figure 3: Principle of frame synchronization

As long as the start position of the FFT window is within region A) no ISI occurs. The only effect suffered by the subchannel symbols is a change in phase that increases with the subcarrier index according to

$$z_{n,l} = a_{n,l} \; H(n,l)\, e^{-j2\pi \frac{k_\epsilon}{N_{fft}}} + n_{n,l} \tag{7}$$

This change in phase cannot be distinguished from the ctf and thus is compensated in the same way.

If the start position is within regions B) the subcarrier symbols are described by [6]

$$\begin{aligned} z_{n,l} = \; a_{n,l} \; & \frac{N_{FFT} - k_\epsilon}{N_{FFT}} \; H(n,l)\, e^{-j\,2\pi\, l\, \frac{k_\epsilon}{N_{FFT}}} \\ & + n_{n,l} + n_{\Delta\epsilon}(n,l) \end{aligned} \tag{8}$$

Due to ISI in addition to the phase rotation the symbol is attenuated and extra noise $n_{\Delta\epsilon}(n,l)$ is introduced since the orthogonality of the system is disturbed. For $k_\epsilon > 0$ a good approximation of the extra noise is given by [6]

$$P(n_{\Delta\epsilon}) = \sum_{i<k_\epsilon} |h_n(i)|^2 E\left\{|a|^2\right\} \frac{2N_{fft} + (k_\epsilon - i)}{N_{fft}^2}(k_\epsilon - i) \tag{9}$$

which proves to be very accurate (a similar expression may be derived for $k_\epsilon < 0$). Equation (9) becomes a minimum if the the energy of the cannel taps inside the guard interval of the receiver time-scale in figure 3 becomes a maximum.

If a coherent system using the channel estimation scheme described above is used k_ϵ not only determines the position of the FFT-window but also implicitly the position of the cir w.r.t. to the interpolation filter. If we assume that $W(i)$ has been designed so that all of the guard interval is within the flat part of the transfer function we can distinguish the same two cases as before. As long as the window start position is within region A) the channel estimation is not affected. But in case B) in addition to the effects described above an offset of the FFT window position will cause channel taps to lie outside the flat part of the interpolation window and the channel estimation will be degraded. In order to analyze the effect of an offset k_ϵ let's consider that $W(i)$ has an ideal raised-cosine transfer function. In the absence of noise the MSE of the channel estimate may be written as a function of the window offset k_ϵ.

$$MSE = \sum_{N_u} \left| H(n,l) - \widetilde{H}_{k_\epsilon}(n,l) \right|^2 \tag{10}$$

If we neglect the effects described by (8) equation (10) yields

$$MSE = \frac{N_u}{N_{fft}} \sum_{i \leq N_h} |h_n(i)|^2 |1 - W(i - k_\epsilon)|^2 \tag{11}$$

Depending on the interpolation filter in most cases the degradation caused by the disturbed channel estimate $\widetilde{H}_{n,l}$ will be bigger than that caused by the ISI.

The above analysis allows to define an appropriate performance criterion. In [7] the BER degradation of the system is used. Since it will be desirable to examine synchronization algorithms in many random channel constellations this criterion is too complex for simulations. In [8] for single carrier systems the portion of channel energy outside a certain window is used as a criterion. Extending this to our problem, we will use as a measure the degradation expressed by formulas (9) and (11), where for each new channel an estimate of the frame position is determined and an approximation according to the equations is calculated. If we define two bounds P_{ISI}^{max} and MSE_{max} an synchronization error occurs if one of the bounds is violated.

For our simulations we consider an OFDM system with $N_{fft} = 2048$ transmitting over a bandwidth of 8 MHz which corresponds to the 2k mode of the DVB-T system. As channel model we take a multipath channel having a profile according to figure 4 which has been measured in the context of DAB. Accordingly the guard-interval is

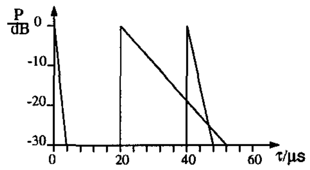

Figure 4: Multiplath delay profile

chosen $\Delta = T_u/4$.

IV Frame Synchronization

For synchronization we have to distinguish between acquisition, where all other relevant system parameters are still unknown and fine synchronization/tracking, where a previous coarse estimate is further improved and adapted to the channel variations.

A Acquisition

During acquisition we have to assume that the carrier frequency offset is still completely unknown. In this case the orthogonality of the system may be disturbed to an extent, where evaluation of information after the FFT is very hard. For a fast and reliable acquisition information in the so called *time domain* should be used.

From single carrier modulation it is well-known that evaluation of periodic information may be used for frame synchronization [9]. In [10] and later in [11] this approach is adopted for the synchronization of OFDM systems.

An estimation of the frame start position can be achieved by searching for periodic structures within the signal. In [9] this is done by minimizing the metric given by (12) that allows to jointly estimate frame-position and frequency offset.

$$\Lambda_p(k_\varepsilon) = \sum_{i=0}^{N_P-1} \left| r(k_\varepsilon + i + N_{PD}) - r(k_\varepsilon + i)\, e^{j\varphi} \right|^2$$
$$\varphi = arg \left\{ \sum_{i=0}^{N_P-1} r(k_\varepsilon + i + N_{PD}) \cdot r^*(k_\varepsilon + i) \right\} \quad (12)$$

N_{PD} is the distance between the periodic portions of length N_P. Note that in OFDM systems the phase may only be used for frequency estimation if post FFT methods allow to resolve resulting ambiguities. Minimizing (12) w.r.t. to the trial parameter

$$\min_{k_\varepsilon} (\Lambda_p(k_\varepsilon)) \quad \rightarrowtail \quad \hat{k}_\varepsilon \quad (13)$$

leads to an estimate for the right position of the FFT window. If the phase is not used it is sufficient to minimize

$$\Lambda_s(k_\varepsilon) = \sum_{i=0}^{N_P-1} |r(k_\varepsilon + i)|^2 + |r(k_\varepsilon + i + N_{PD})|^2 - 2\left| \sum_{i=0}^{N_P-1} r(k_\varepsilon + i + N_{PD}) \cdot r^*(k_\varepsilon + i) \right| \quad (14)$$

which is completely independent of the frequency offset. (14) may be transformed into a correlation metric as in [12, 11]. The general characteristics of the metric are depicted in figure 5. The metric does not have a single minimum

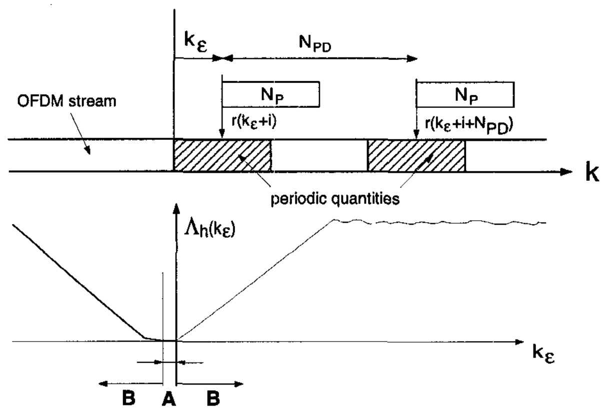

Figure 5: Periodicity metric

but a flat part that corresponds to the ISI-free part A) of the guard interval. Any minimum of the metric detected within A) will yield a valid position for the FFT-window.

In [12] the periodicity inherent in the guard interval is used for synchronization. Since the guard interval usually is affected by ISI the result of the estimation depends on a priori assumptions about the channel. Here we will consider an ISI free periodic portion of the OFDM signal which may be provided by either using longer guard intervals for some OFDM symbols dedicated for synchronization, or by using periodic OFDM symbols as in [6, 11]. To minimize synchronization overhead symbols with a reduced number of subcarriers, still carrying random data may be used for this purpose.

For our simulations we distinguish between two transmission scenarios with different requirements. For a *continual transmission* such as terrestrial TV-broadcasting the requirements for the accuracy of the channel estimate and for the frame-error-rate are very high. In order to avoid an error floor due to the frame-estimation a $MSE_{max} = -E_s/N_0 - 10dB$ and a quasi error-free synchronization should be provided. Since acquisition time is not so critical in this context, the estimation may be improved by averaging over several consecutive metrics $\Lambda_s(k_\varepsilon)$. In a *burst-oriented transmission*, the requirements for the accuracy of the channel estimation will usually be lower. Furthermore a comparably high frame-error-rate is tolerable due to the use of ARQ techniques [13]. Since the requirements for acquisition time are higher in this case synchronization should be achievable with a single preamble.

Figures (6) and (7) show our simulation results for different MSE_{max} and N_P. We have used prolonged guard intervals of length $\Delta + N_P$. The distance between the periodic portions of the signal then is $N_{PD} = N_{fft}$. The interpolation filter used for channel estimation is a Wiener filter optimized for a cir of maximum delay Δ and $E_s/N_0 = 20dB$.

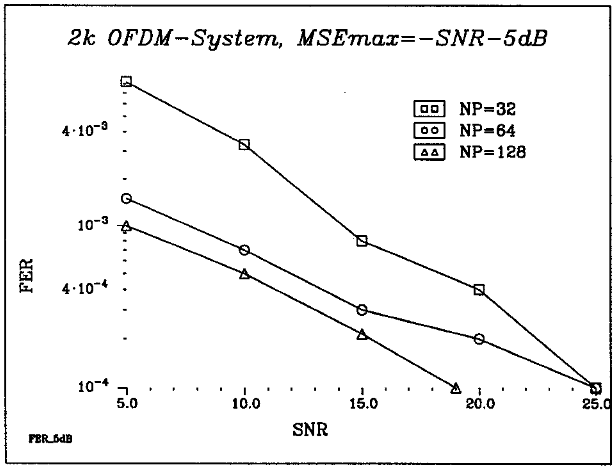

Figure 6: Frame error rate if 5dB degradation of the channel estimate are permitted

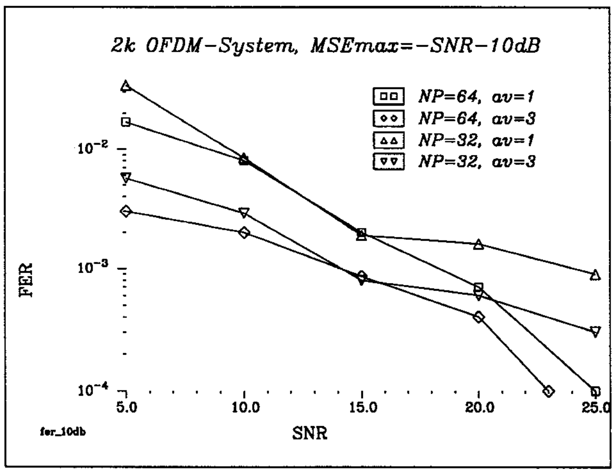

Figure 7: Frame error rate if 10dB degradation of the channel estimate are permitted

As a comparison of figures 6 and 7 shows the frame error rate increases dramatically as MSE_{max} decreases. Increasing N_p improves results only to a certain extent. For big windows the FER even increases. Averaging over several metrics improves the estimation considerably. Still it is not possible to provide a quasi error-free synchronization. In some channel constellations i.g. if there are single path of low energy the probability of an error becomes high. Figure 8 shows a metric that is typical for such a channel. Since it is very flat around the actual synchronization point, the reliable detection of a minimum is hard.

For a burst oriented transmission the scheme presented can provide a sufficient frame synchronization. Since the ISI is compensated by the guard interval which is needed anyway the amount of additional bandwidth for training data is very small. For a continual scenario the synchronization is not sufficient for all channel constellations. In this case we need further processing to improve and track the frame start position.

B Tracking

During tracking we can assume that frequency synchronization has been achieved already. Therefore post-FFT algorithms may be used to first improve the coarse estimate generated by pre-FFT methods and then follow movements of the cir w.r. to the FFT-window that are caused by channel variations and a possible sampling clock offset.

Since the channel-estimation is affected most severely by a frame-offset it may be used for synchronization. Obviously as the energy of the cir within the interpolation window $W(i)$ is maximized, so is the energy of the channel estimate. Since no further information is added by the interpolation it is sufficient to evaluate the estimate at the pilot positions. The resulting metric is given by

$$\Lambda_H = \sum_{l\epsilon P} \left|\widetilde{H}(n,l)\right|^2$$
$$\max_{k_\epsilon}\left(\Lambda_H(k_\epsilon)\right) \quad \mapsto \quad \hat{k}_\epsilon \qquad (15)$$

Extending the approach in [8] to the case of OFDM this metric can be motivated by the maximum likelihood theory. The full ML metric then is given by

$$\Lambda_T = \sum_{l\epsilon P} \left|z_{n,l} - p_{n,l}\cdot \widetilde{H}(n,l)\right|^2 \qquad (16)$$

which implicitly is also considers the ICI effects.

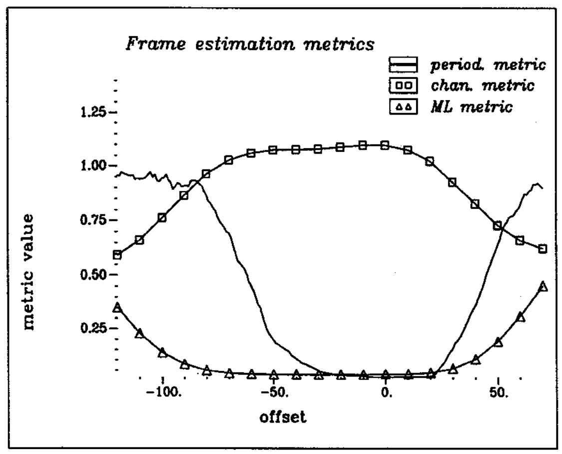

Figure 8: Metrics used for frame synchronization

If a Wiener filter is used for interpolation both metrics implicitly benefit from the a priori information used for the optimization of the interpolator. Figure 8 shows the metrics if the same interpolation as above is used. The resulting metrics are very smooth and allow a reliable detection of the synchronization point even under very noisy conditions. Since (16) also contains the unfiltered pilot symbols it is more vulnerable to channel noise. Our simulation results indicate that both metrics provide very robust frame synchronization. Within the scope of our simulations no frame-error has been detected for a $MSE_{max} = -E_s/N_0 - 10dB$. Looking at the absolute values of the estimated frame position (15) yields better results. In particular at low SNR.

Since both metrics make use of the channel-estimation units the extra complexity introduced is moderate. When the algorithm is used to improve a previous coarse estimate it may be implemented in a feed-forward structure. The metric may then be maximized by a search, varying the position of the FFT-window. In the case of a continual transmission for which this scheme is to apply the resulting additional acquisition time is acceptable. For tracking mode the algorithm may be realized within a tracking loop triggering the sampling clock. The behavior of such a loop in particular in a mobile environment remains subject to further investigation. In this case it may be necessary to use the metric according to equation (16) which is less sensitive to channel variations.

V Discussion and Conclusion

For systems based on OFDM the impact of a frame-misalignment has been discussed. Analysis shows that channel estimation via interpolation as it is commonly used for coherent OFDM systems is very sensitive to a frame misalignment. It follows that for such systems the requirements for the accuracy of the frame estimation are much higher than for systems using differential modulation. An appropriate performance-criterion for frame-synchronization algorithms has been derived. Using this criterion the possibility of frame-synchronization via evaluation of periodic structures within the OFDM-signal has been examined. Simulations show that for a burst-oriented transmission robust frame synchronization is possible with low synchronization overhead. Only if the requirements to the accuracy of the estimate and the frame error rate are extremely high, such as in terrestrial TV transmission, the algorithm fails under some channel conditions. A second algorithm that allows reliable synchronization under these conditions has been presented which is based on the evaluation of the channel-estimation. Simulations show that it can provide a reliable estimate under all channel-conditions. The implementation and behavior of this algorithm within a tracking loop will be subject to further investigation.

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