Joint Frame Synchronization and Frequency Offset Estimation in OFDM System

Zhongshan Zhang
Wireless Communication Center
Beijing University of Posts and Telecommunications
Beijing, China
adhoc@bupt.edu.cn

Ming Zhao
Wireless Communication Center
Beijing University of Posts and Telecommunications
Beijing, China
manyzhao@163.com

Abstract—**One new joint frame synchronization and carrier frequency offset estimation method for orthogonal frequency-division multiplexing (OFDM) system is proposed. The same training-symbol-block (TSB) is needed for both frame synchronization and carrier frequency offset estimation. The carrier frequency offset estimation including acquisition and tracking, and acquisition can be further divided into Pre-Acquisition and Fine Acquisition. As soon as frequency offset acquisition finished, timing synchronization is also performed at the same time. The acquisition range is as large as one half of overall signal bandwidth. The theoretical variance error lower bound for our frequency offset tracking algorithm is also derived in this paper.**

Keywords- synchronization; frequency offset estimation; OFDM

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is an effective transmission scheme to combat multipath fading [1-2]. By inserting a guard interval between symbols blocks called cyclic prefix, the intersymbol interference (ISI) can be mitigated. OFDM was adopted as the modulation scheme for a DAB (Digital Audio Broadcasting) system [3] and ADSL (Asymmetry Digital Subscriber Loop) [4] and was also proposed as the terrestrial HDTV transport in Europe [5].

Proper synchronization scheme should be provided for OFDM systems to decide the start point of FFT window at receiver so as to demodulate the transmitted signal correctly. If FFT window start point is not the first sample of one OFDM symbol but it locates within the cyclic prefix, some phase rotation appears in the demodulated signal, which can be corrected after channel estimation; but if the start point locates outside of the cyclic prefix, ISI occurs. Many frame synchronization schemes have been presented [6-16]. Data-aided synchronization scheme is based on the special synchronization symbols periodically inserted at the transmitter to get frame clock [7][11][13]. This kind of synchronization schemes can get high estimation accuracy, but the insertion of training symbols will decrease the system capacity. Some nondata-aided frame synchronization schemes based on the cyclic prefix have been presented [9][10][12]. This kind of scheme only uses the in-phase and the quadrature sign bits of the OFDM symbol to perform symbol synchronization, but its performance is easily affected by multipath signals. Method provided in [15] needs no pilot symbol and cyclic-prefix, it explores the cyclostationarity in the received signal for blind synchronization.

OFDM systems are also very sensitive to carrier frequency offset, and frequency offset with even a small fraction of subcarrier spacing will degrade the performance of OFDM receiver greatly [17]. Many data-aided frequency offset estimators are presented [11][13][17-19]. Some nondata-aided estimator using the cyclic prefix has the acquisition range limited within ± 0.5 subcarrier spacing [9]. There are also some methods for increasing acquisition range [10][13][17].

In this paper, a new data-aided joint frame synchronization and carrier frequency offset estimation method for orthogonal frequency-division multiplexing (OFDM) system is proposed. Different from previous methods, the new proposed scheme performs frame synchronization and carrier frequency offset acquisition at the same time, not sequentially. After frequency offset acquisition, the remaining frequency offset will be further corrected by tracking algorithm with high accuracy. The same TSB is need for both frame synchronization and carrier frequency offset estimation, and the carrier frequency offset acquisition range is as large as one half of overall signal bandwidth.

This paper is organized as follows. Section II gives a brief overview of OFDM fundamentals. A new frame synchronization estimator will be proposed in Section III, and a new carrier frequency offset correction scheme will be proposed in Section IV and V, followed by the simulation results given in Section VI. Conclusions are drawn in Section VII.

II. OFDM FUNDAMENTALS

OFDM input signals are parallel complex numbers from some signal constellation (for example, PSK or QAM). After IDFT, cyclic prefix is added; after parallel-to-serial (P/S) conversion, the time domain signals will be transmitted. At the receiver, without considering of the effect of channel attenuation, there is a simple relationship between the transmitted and the received signals:

$$r(k) = s(k-l)e^{j2\pi\varepsilon k/N} + n(k) \tag{1}$$

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where N is the DFT length, ε is the carrier frequency offset normalized to subcarrier spacing, $n(k)$ is the sample of zero mean complex *Additive White Guassian Noise* (AWGN) process, and l is the timing offset.

III. FRAME SYNCHRONIZATION

Our new proposed synchronization algorithm is data-aided, and the TSB in time domain contains two training symbols with equal length, and the second symbol is the inverse-sequence repeat of the first one. The form of TSB is: $S = [s(0)\ s(1) \ldots s(N\text{-}1)\ s(N\text{-}1) \ldots s(1)\ s(0)]$.

This TSB can be used for frame synchronization. Our timing metric for frame synchronization is:

$$M(d)=\left|\sum_{k=d}^{d+N-1} r(2N-1-k)\,r^{*}(k)\right| \tag{2}$$

Without loss of generality, the start position index of a TSB is assumed to be 0, and the probability of correct synchronization is:

$$p\{M(0)>M(d)\,|\,\forall d\neq 0\}=$$

$$1+\sum_{k=1}^{2(N-1)} C_{2(N-1)}^{k}\cdot(-1)^{k}\cdot\frac{1+\frac{2}{SNR}+\frac{1}{SNR^{2}}}{1+\frac{2(k+1)}{SNR}+\frac{(k+1)}{SNR^{2}}}\cdot\exp\left\{-\frac{k\cdot\frac{\sin^{2}(2\pi\varepsilon)}{\sin^{2}(2\pi\varepsilon/N)}}{N\left[1+\frac{2(k+1)}{SNR}+\frac{(k+1)}{SNR^{2}}\right]}\right\} \tag{3}$$

Because the effect of noise, some synchronization errors may occur to the new proposed estimator. The mean square error for the proposed frame synchronization estimator is derived as:

$$MSE(N,SNR,\varepsilon)=\sum_{d=-(N-1)}^{N-1} d^{2}\cdot\frac{1-p\{M(0)>M(d)\,|\,\forall d\neq 0\}}{2(N-1)}$$

$$=\frac{N(2N-1)}{6}\cdot\sum_{k=1}^{2(N-1)} C_{2(N-1)}^{k}\cdot(-1)^{k+1}\cdot\frac{1+\frac{2}{SNR}+\frac{1}{SNR^{2}}}{1+\frac{2(k+1)}{SNR}+\frac{(k+1)}{SNR^{2}}}\cdot\exp\left\{-\frac{k\cdot\frac{\sin^{2}(2\pi\varepsilon)}{\sin^{2}(2\pi\varepsilon/N)}}{N\left[1+\frac{2(k+1)}{SNR}+\frac{(k+1)}{SNR^{2}}\right]}\right\} \tag{4}$$

From equation (4) it is shown that $MSE(N,SNR,\varepsilon)$ is a function of N, SNR and ε. Note that in this paper, N is only used to indicate the DFT length of the *training symbols*, not for data symbols. Figure 1 shows $MSE(N,SNR,\varepsilon)$ with DFT length of 16 and with SNR equals to 5dB. It is shown in figure 1 that the mean square error of the proposed frame synchronization algorithm is very sensitive to carrier frequency offset. Correct frame synchronization can be only performed within very small frequency offset range (within a main lobe in figure 1); outside of this range, the mean square error will increase quickly.

The minimum mean square error (MMSE) is got when ε equals to 0:

$$MMSE(N,SNR)$$

$$=\frac{N(2N-1)}{6}\cdot\sum_{k=1}^{2(N-1)} C_{2(N-1)}^{k}\cdot(-1)^{k+1}\cdot\frac{1+\frac{2}{SNR}+\frac{1}{SNR^{2}}}{1+\frac{2(k+1)}{SNR}+\frac{(k+1)}{SNR^{2}}}\cdot\exp\left\{-\frac{k\cdot N}{1+\frac{2(k+1)}{SNR}+\frac{(k+1)}{SNR^{2}}}\right\} \tag{5}$$

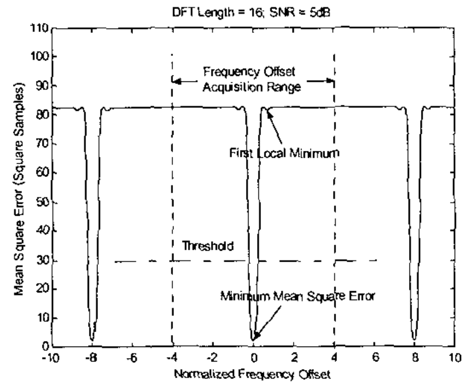

Figure 1. Mean Square Error of frame synchronization with different carrier frequency offset

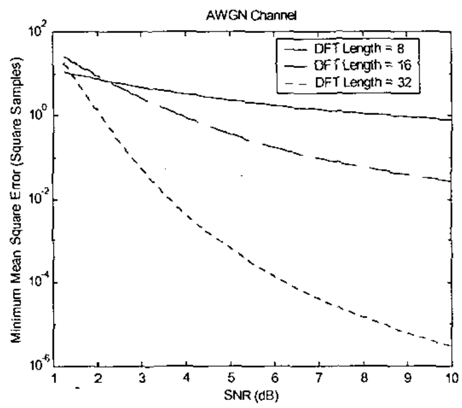

Figure 2. Minimum Mean Square Error with different N and SNR

Without frame synchronization, the start position of training symbols can't be located, and frequency offset can't be estimated correctly; before frequency offset acquisition, frame synchronization will be not performed. The frame synchronization and carrier frequency offset correction can not be performed separately. This is just the key character of the new proposed joint frame synchronization and frequency offset estimation scheme. When SNR is large enough (larger than 3dB), with the increase of DFT length N and SNR, the MMSE will decrease accordingly, which is illustrated in Figure 2. The judgement of correct timing synchronization may be as follows: select a threshold which should be larger than MMSE (but should not be too large), if frame synchronization estimator's variance error is smaller than that threshold, frame synchronization is assumed to be performed and the remaining frequency offset is assumed to be within the tracking range. Larger N and higher SNR imply smaller MMSE, which make the threshold easier to be selected.

IV. FREQUENCY OFFSET ACQUISITION

At the start of acquisition, frequency offset is fully unknown to the receiver, and timing synchronization is not performed yet. First $(m+1)$ $(m \geq 2)$ TSBs (each TSB should be different from its neighbors) are transmitted. At the receiver, these TSBs are buffered. It is shown in figure 1 that $MSE(N, SNR, \varepsilon)$ is periodic with period of $N/2$, so that the acquisition range of the proposed acquisition algorithm is limited within $\pm N/4$ times subcarrier spacings. The acquisition can be sub-divided into two parts: Pre-Acquisition and Fine Acquisition, as shown in Figure 3.

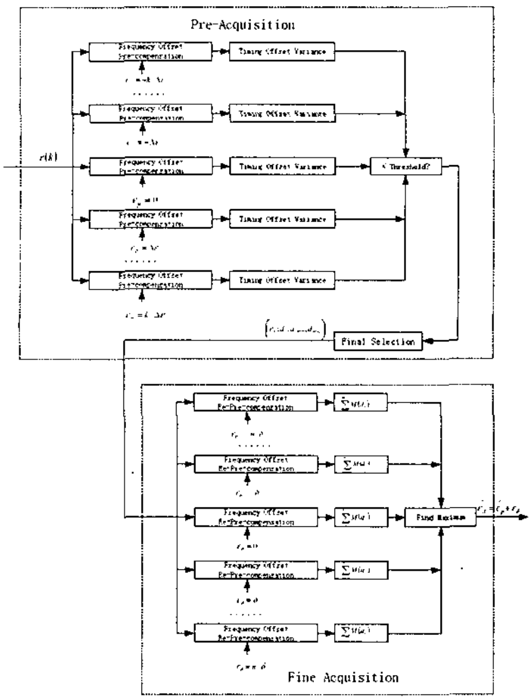

Figure 3. Frequency offset acquisition

For acquisition, $(2k+1)$ different $\hat{\varepsilon}_p$ values ($-k\Delta\varepsilon, -(k-1)\Delta\varepsilon, \ldots 0, \ldots (k-1)\Delta\varepsilon, k\Delta\varepsilon$) are used to pre-compensate for the carrier frequency offset of the buffered training symbols, which results in $(2k+1)$ training sequences with different remaining frequency offset. The pre-compensation range is $-k \cdot \Delta\varepsilon \leq \hat{\varepsilon}_p \leq k \cdot \Delta\varepsilon$ where $0 < \Delta\varepsilon < 0.5$ and $k \cdot \Delta\varepsilon < N/4$. For each compensated training sequence, timing metric $M(d)$ is used to find the start of each TSB (without loss of generality, we can represent those estimated start positions of n^{th} ($-k \leq n \leq k$) training sequence as $d_{n,0}, d_{n,1}, \ldots, d_{n,m-1}$). *Timing Offset Variance* (TOV) of n^{th} training sequence is calculated as $TOV(n) = \sum_{i=0}^{m-2} \frac{(d_{n,i+1} - d_{n,i} - 2N)^2}{m-1}$ which can be used to approximately represent the mean square error of the proposed frame synchronization estimator for that training sequence. The $(2k+1)$ TOV values may be different from each other. The $\hat{\varepsilon}_p$ corresponding to the minimum TOV is selected as the output of Pre-Acquisition. The training sequence corresponding to that minimum TOV is also the output of Pre-Acquisition, and the start positions for each TSB in that sequence can be represented as $d_0, d_1, \ldots, d_{m-1}$. That sequence will be used in Fine Acquisition.

After Pre-Acquisition, the remaining normalized frequency offset $\varepsilon - \hat{\varepsilon}_p$ will be within ± 0.5 subcarrier spacing. From the following section we may know that the tracking range of the proposed tracking algorithm is $\pm \frac{N}{2(2N-1)}$ times subcarrier spacing, and the remaining frequency offset $\varepsilon - \hat{\varepsilon}_p$ may be beyond the tracking range, so that Fine Acquisition is needed for further frequency offset correction.

Fine Acquisition is very similar to Pre-Acquisition. A number of $\hat{\varepsilon}_\delta$ values are selected to re-pre-compensate for the remaining frequency offset of the training sequence with minimum TOV. The re-pre-compensation range is $-n \cdot \delta \leq \hat{\varepsilon}_\delta \leq n \cdot \delta$ where $0 < \delta < \frac{N}{2(2N-1)}$ and $n \cdot \delta < 0.5$. With each re-pre-compensated sequence, $\sum_{i=0}^{m-1} M(d_i)$ is computed, and the training sequence that maximum $\sum_{i=0}^{m-1} M(d_i)$ has the remaining frequency offset nearest to zero, and the $\hat{\varepsilon}_\delta$ corresponding to this sequence is the final output of Fine Acquisition.

V. FREQUENCY OFFSET TRACKING

After Fine Acquisition, there will also exist some remaining frequency offset to be further corrected. If the remaining frequency offset equals to zero, $M(0)$ will get its maximum value. The tracking algorithm can be illustrated as:

$$\hat{\varepsilon} = \frac{N \sum_{k=0}^{N-1} \sum_{m>k}^{N-1} \left| r(2N-1-k) r^*(k) \right| \cdot \left| r(2N-1-m) r^*(m) \right| \cdot (m-k) \cdot \left[angle\{ r(2N-1-k) r^*(k) \} - angle\{ r(2N-1-m) r^*(m) \} \right]}{4\pi \sum_{k=0}^{N-1} \sum_{m>k}^{N-1} \left| r(2N-1-k) r^*(k) \right| \cdot \left| r(2N-1-m) r^*(m) \right| \cdot (m-k)^2}$$

$$(6)$$

The tracking error of this tracking estimator is:

$$e \approx \frac{\sum_{k=0}^{N-1}\sum_{m=k}^{N-1}\left|r(2N-1-k)r^*(k)\right|\cdot\left|r(2N-1-m)r^*(m)\right|\cdot(m-k)\cdot\left[(2N-1-2k)e_k-(2N-1-2m)e_m\right]}{2\sum_{k=0}^{N-1}\sum_{m=k}^{N-1}\left|r(2N-1-k)r^*(k)\right|\cdot\left|r(2N-1-m)r^*(m)\right|\cdot(m-k)^2} \quad (7)$$

where:

$$e \cong \frac{Nr\sin\left(\theta-\left(\frac{2\pi\varepsilon(2N-1-2i)}{N}\right)\right)}{2\pi(2N-1-2i)\left[\left|S(i)\right|^2+r\cos\left(\theta-\left(\frac{2\pi\varepsilon(2N-1-2i)}{N}\right)\right)\right]} \quad (8)$$

for high SNR and $\theta \in [-\pi \quad \pi]$.

Because $E\left\{e\,\middle|\,|\varepsilon|<\frac{N}{2(2N-1)}\right\}=0$, this estimator is conditional unbiased, and the Cramer-Rao lower bound of its variance error is:

$$Var\left\{e\,\middle|\,|\varepsilon|<\frac{N}{2(2N-1)}\right\} \geq \frac{3N}{4\pi^2\left(4N^2-1\right)SNR} \quad (9)$$

In order to make the new proposed frequency offset tracking algorithm work correctly, it is should be guaranteed that $2\pi\varepsilon\cdot\frac{2N-1}{N}<\pi$, *i.e.*, the tracking range is limited within $\pm\frac{N}{2(2N-1)}$ subcarrier spacing. For large N values, the tracking range nearly equals to $\pm 1/4$ subcarrier spacing.

VI. Simulation Results

All estimators in the new proposed scheme are derived in AWGN channel, and their performances will be evaluated in multipath channel. In multipath channel, the received signal can be represented as $r(i)=\sum_{k=1}^{N_p}\alpha_k s(i-\tau_k)+n(i)$, where α_k denotes the k^{th} path complex gain, τ_k denotes the k^{th} path time delay in samples, and N_p is the total number of paths. In this paper, we assume that $\tau_1=0$ which corresponds to the maximum power tap, and other taps are interference signals to the first path and can be seen as interference noise, which results in the reduction of effective signal-to-interference-noise ratio (SINR).

In this paper, a wireless system is assumed operating at $5GHz$ and with bandwidth of $10MHz$. It is also assumed that the Maximum Doppler Shift is $231.48Hz$. An outdoor dispersive, fading environment is chosen: The channel has an exponentially decaying power delay profile with root mean square width equal to $0.1\mu s$ and a maximum delay spread of $1.2\mu s$. It is modeled to consist of 4 independent Rayleigh-fading taps and additive noise. The TSB length is assumed to be 256 (not including the cyclic-prefix).

The proposed tracking algorithm works better on smaller frequency offset condition than on larger frequency offset condition; at high SNR, that kind of performance difference will disappear, which illustrated in figure 4. The proposed tracking algorithm works well in AWGN channel; in multipath channel, since multipath signals may reduce the effective SINR, which results in large tracking errors. It is shown in figure 4 that as the increases of SNR, a performance floor appear to the proposed tracking algorithm.

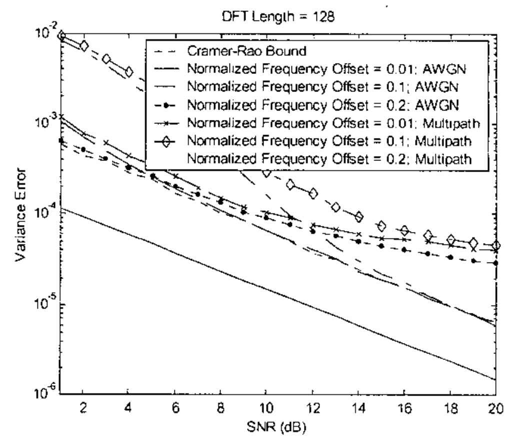

Figure 4. . Variance Error of frequency offset tracking and its Theoretical Lower Bound

VII. Conclusions

Data-aided synchronization algorithm may result in the decreases of system capacity because of the periodically inserted training symbols. The synchronization scheme proposed in this paper perform frame synchronization and carrier frequency offset estimation using the same training symbols, so that the synchronization cost can be reduced to some extent. When acquisition, $\Delta\varepsilon$ should be selected as large as possible in order to finish frequency offset Pre-Acquisition with high speed and with low computational complexity, and small δ should be used to make the remaining frequency offset after Fine Acquisition well within the tracking range. In fact, in Pre-Acquisition, $\Delta\varepsilon$ can be set much smaller than $\frac{N}{2(2N-1)}$ to guarantee the remaining frequency offset be not beyond the tracking range, and as a result, no Fine Acquisition needed, which will shorten the acquisition time greatly at the cost of increasing the computational complexity.

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