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An Investigation of Dynamic Sub-carrier Allocation in MIMO-OFDMA Systems

Ying Peng, Simon Armour and Joseph McGeehan

***Abstract*—In this paper, Orthogonal Frequency Division Multiple Access (OFDMA) systems with dynamic deterministic (as opposed to pseudo-random) allocation of sub-carriers to users to exploit multiuser diversity are investigated. Previously published work on dynamic multi-user subcarrrier allocation [1,2] for OFDMA systems with SISO channels is surveyed. A near-optimal, low complexity algorithm for SISO systems [3], which is structurally similar to the algorithm in [2], is extended to the case of MIMO systems in this paper. The optimality and adaptability of this algorithm are analysed by formulating an assignment problem and comparing with one optimal and two extended sub-optimal strategies proposed based on previous work. Consideration of a MIMO channel creates further issues for the sub-carrier allocation process. In particular, methods whereby an appropriate sub-carrier allocation may be exploited to minimize the effects of correlation in MIMO channels is of considerable interest. Several novel variants of the algorithm (referred to as 'schemes') are proposed and evaluated for MIMO systems employing both Space-Time Block Coding (STBC) and Spatial Multiplexing (SM) in both un-correlated and correlated fading channels. Simulation results identify the most suitable schemes for both STBC and SM and in particular show that substantial improvements in performance (in terms of BER) in correlated channels can be achieved by means of suitable sub-carrier allocation. In uncorrelated channels, the best scheme can offer approximately 7dB gain over the conventional MIMO channel; in highly correlated channels, even more substantial improvements (>11dB gain for STBC, >20dB gain for SM) in performance can also be achieved, demonstrating the ability of a well designed sub-carrier allocation scheme to mitigate the debilitating effects of correlation on MIMO systems.**

***Index Terms*—OFDMA, Sub-carrier allocation, SISO, MIMO, STBC, SM**

I. Introduction

ORTHOGONAL Frequency Division Multiple Access (OFDMA), also referred to as multiuser-OFDM, is an extension of Orthogonal Frequency Division Multiplexing (OFDM), and is a highly regarded candidate modulation and multiple access method for a 4th generation physical layer (4G PHY). It has recently been chosen for the IEEE 802.16 and DVB-RCT standards [4,5] and received significant research interest. Similarly to OFDMA, various combinations of Direct-Sequence Code-Division Multiple-Access (DS-CDMA) [6] and OFDM – e.g. Multi-Carrier Code-Division Multiple-Access (MC-CDMA) [7] – are also highly regarded. In practice, there is little difference between the schemes in their robustness against noise and inter-cell-interference [8]. However, OFDMA can often outperform the others for high system loads due to its lower complexity and ability to maintain orthogonality on frequency-selective fading channels.

OFDMA makes use of OFDM modulation whilst allowing multiple-access by separating symbols in frequency and optionally in time as well. In a certain time slot, all or some of the active sub-carriers can be used by a given terminal. OFDMA therefore concentrates uplink transmit power into a 'sub-channel' which consists of 1-*N*th of the whole bandwidth (where N is the number of terminals), resulting in a $10\log_{10} N$ dB gain.

Further to this uplink gain, there is also the opportunity to exploit the advantages of multiuser diversity to mitigate channel fading. This is the focus of the algorithms proposed in [1,2,3]. In a frequency selective channel, sub-carriers will perceive a large variation in channel gain and the perceived channel will be different for each user. If a deterministic rather than random allocation of sub-carriers is employed, multi-user diversity can be exploited. In this way, the majority of sub-carriers allocated to each user perceive gain (relative to the mean for all frequencies) rather than attenuation in the radio channel. Such a deterministic allocation algorithm will be referred to here as a Dynamic Sub-Carrier Allocation (DSA) algorithm.

DSA algorithms have been proposed previously in [1,2,3] for the case where the channel on a per link basis is Single Input Single Output (SISO). [3] is of particular relevance insofar as it is a low complexity, adaptive, multi-user DSA algorithm in which the per-sub-carrier channel gain of each user is used as the metric to allocate the sub-carriers and ensure a fair allocation for all users insofar as they all receive approximately equal gain from the DSA whilst not adversely affecting overall system capacity. However, whilst achieving significant gain, this algorithm might not reach the optimal solution (in terms of achieving the total maximum channel gain for users). The Hungarian Method [9] and Sort-Swap algorithm [10] are introduced and extended to solve this allocation problem as an

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assignment problem to maximizing total channel gain and achieve an optimal solution. However simulation results show that the DSA algorithm in [3] achieves perceived channel gains within a fraction of a dB of the optimal solution [11].

The core novelty of this paper lies in the fact that it considers another opportunity to exploit diversity in addition to the uplink and multiuser diversity gains described above. This opportunity is specific to MIMO systems wherein the flexibility of sub-carrier allocation facilitates an opportunity to mitigate the debilitating effects of correlation in the channel.

Considering MIMO systems, the DSA algorithm [3] is initially extended to three candidate schemes (schemes 1, 2 and 3) – all MIMO compatible – and performance is evaluated in un-correlated channels. The schemes inherit the low complexity of the algorithm in SISO and get much benefit from spatial diversity and multi-user diversity in un-correlated channel scenarios.

However, as is well known, MIMO system capacity mostly depends on the spatial correlation properties of the radio channel. An obvious way to achieve de-correlation between a set of antenna elements is to place them far away from each other. However, in most cases, the nature of the equipment will limit the antenna spacing. The nature of the environment may also limit the effectiveness of this method, for example due to the keyhole effect [12]. Two further adaptive multi-user sub-carrier allocation algorithms extended from [3] (schemes 4 and 5) are proposed in this paper. These schemes are designed specifically to combat the debilitating effects of correlation whilst still seeking a maximal or near-maximal allocation of channel energy. The effectiveness of these methods is evaluated in both un-correlated and correlated channels.

This paper is organized as follows: section II describes the system model used in this paper. Section III summarizes the DSA algorithm for the SISO OFDMA system and confirms its near-optimality by analyzing its performance in comparison with optimal and alternative sub-optimal solutions. The extended schemes for un-correlated and correlated scenarios are proposed in section IV and V respectively. Section VI introduces the simulation environment and parameters. Section VII includes all simulation results and performance analysis. Conclusions are provided in section VIII.

II. System Model

The OFDMA system considered here consists of one Base Station (BS) and multiple Mobile Stations (MSs) all of which possess either two or four antennas. In this paper, the downlink is considered for the sake of simplicity. However, the DSA algorithm and the diversity gains which it achieves are, in principle, equally applicable to the uplink (which will enjoy the further benefits of OFDMA uplink gain as discussed above). The BS is considered to communicate simultaneously with multiple MSs, each of which is allocated a single sub-channel consisting of a given number of OFDMA sub-carriers (equal numbers of sub-carriers per sub-channel and a single sub-channel per MS is assumed for simplicity in this paper but this is not essential to the functionality of the DSA algorithm).

The BS takes the downlink data for all MSs and applies independent bit level error control coding, symbol mapping and serial to parallel conversion. In the case of SISO, a DSA mapping process allocates the parallel data symbols to appropriate sub-carriers as indicated by the DSA algorithm. On the other hand, in the case of MIMO, each user signal is subject to either Space-Time Block Coding (STBC) or Spatial Multiplexing (SM) at the BS. The resultant 2 or 4 (depending on the number of transmit elements) coded/multiplexed user symbol streams are assigned to appropriate sub-carriers as indicated by the DSA algorithm. Subsequently, the symbols are OFDM-modulated via Inverse Fast Fourier Transform and a Guard Interval (GI) is inserted between symbols to avoid inter-symbol interference (ISI). The corresponding block diagrams are shown in Fig. 1 and 2.

The signal received by the MSs takes the form of the BS signal, convolved with the channel transfer function matrix and additionally subject to AWGN. Symbols sent by the BS to other MSs (users) may be discarded after the FFT in the MS receiver.

After extracting the GI and applying a Fast Fourier Transform process at each MS receiver, the sub-carriers assigned to a given MSs sub-channel are extracted according to the allocation of sub-carriers determined by the DSA algorithm (the method by which this allocation is communicated to the MS is not considered in this paper). Then, for an STBC system, a combining scheme [13] (proposed by Alamouti) is applied. In the SM case, a minimum mean-squared error (MMSE) receiver is used to balance multi-Stream Interference (MSI) mitigation with noise enhancement and minimize the total error [14]. Finally, the signals by zero-forcing process (in the case of SISO) or the combined (STBC) or demultiplexed (by MMSE in SM) signals are subject to Forward Error Correction (FEC). In this paper, convolutional encoding, CSI enhanced soft Viterbi decoding and block interleaving are assumed as a standard case but the DSA algorithm is again independent of such issues.

Three graphs of the system model of SISO and MIMO cases are shown in Fig. 1 and 2 ((a) and (b)) to indicate the whole process mentioned above. The later two also can be extended to higher dimension MIMO system. Note that all connections to the channel estimator and the control channel between the DSA algorithm and the receiver are shown dashed since neither channel estimation nor the communication of control information is considered explicitly in this paper. In [15], the impact of non-ideal channel estimation is considered in detail in the context of the DSA algorithm and it is shown that the proposed algorithm is insensitive to channel estimation errors. The exchange of control information has received fairly limited attention in the open literature [26, 27].

III. The DSA in SISO-OFDMA

The SISO DSA algorithm presented in [3] is considered as a starting point here. Although structurally similar to the

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algorithm in [2] (both exhibit relatively low complexity), this

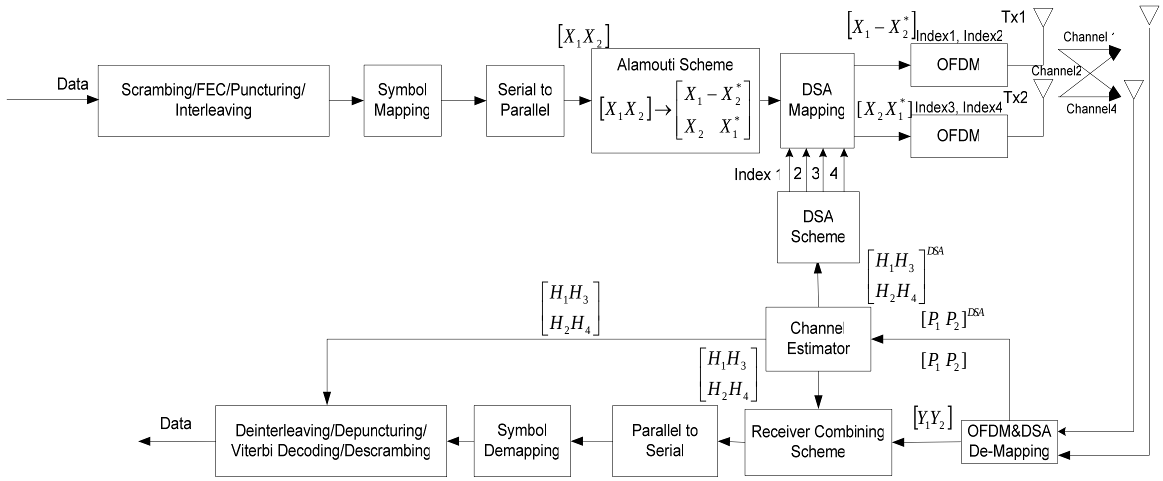
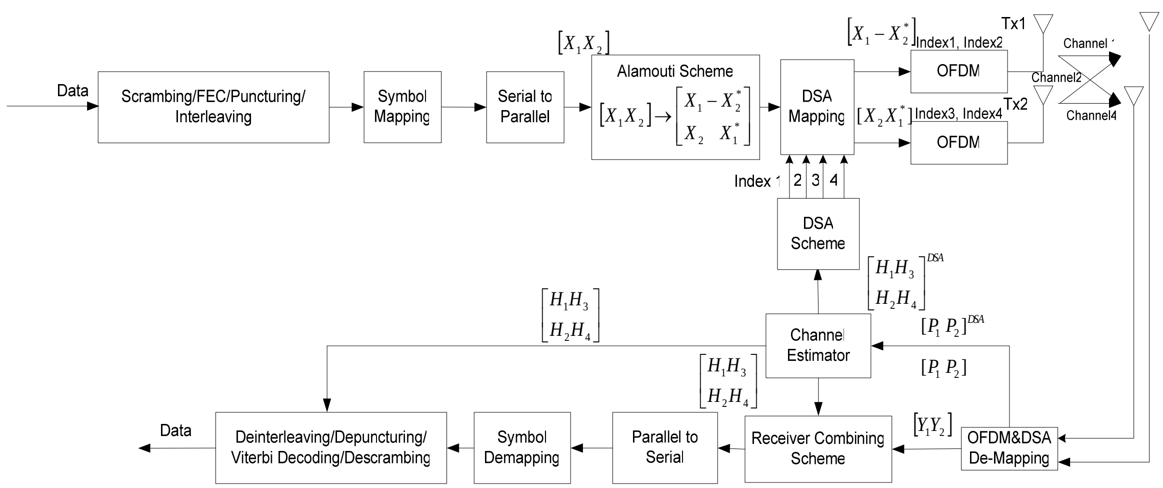
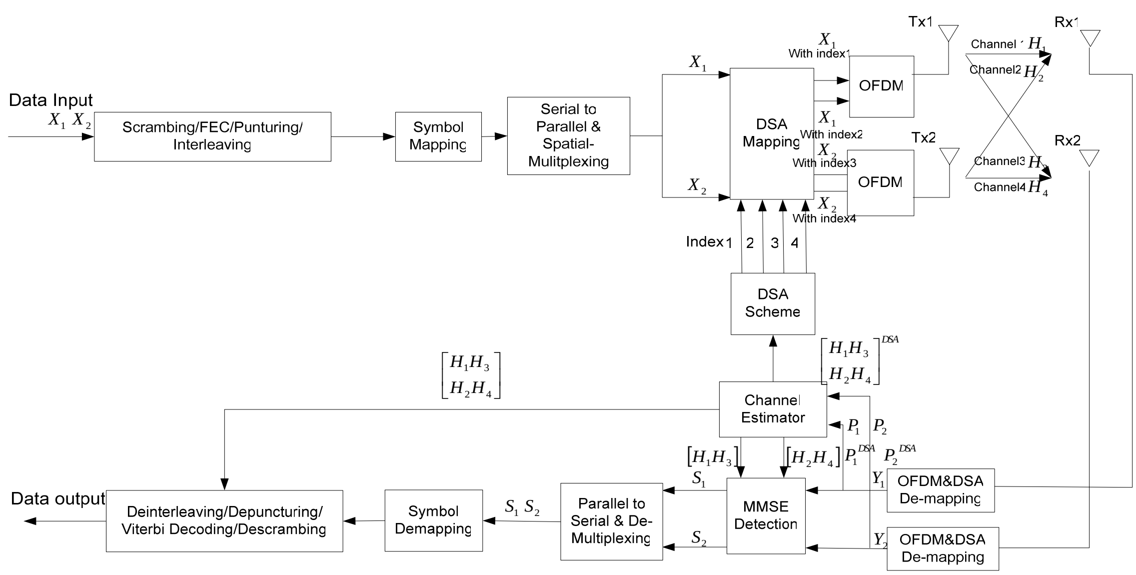


Fig. 1. System model: DSA-OFDMA in SISO system



(a)



(b)

Fig. 2. (a) STBC 2X2 OFDMA system model (b) SM 2X2 OFDMA system model

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algorithm considers the channel gain of each user as the metric to allocate the sub-carriers and ensures a 'fair' allocation for all users by allocating an equal number of sub-carriers to all users. In order to investigate the optimization probability of this algorithm, the classic assignment problem based on channel gain as the metric is first defined and some relative solutions such as an extended Hungarian method, 'OCSA', sub-optimal solutions, 'MGSS', 'DSA' algorithm and improved 'DSA' algorithm 'DSA-swap' are presented and compared.

A. Assignment problem for optimum solution

Assuming in an OFDMA system, provision of QoS and data rate requests have been fixed, the number of sub-carriers is specified by the request of submitted data rate for each user *k*. $h_{k,n}$ is the channel response for user k, sub-carrier *n* for a certain channel. $|h_{k,n}|$ manifests the relative channel transfer function amplitude. The channel gain matrix $H = \{|h_{k,n}|^2\}$, $k = 1,2,\ldots,K, n = 1,2,\ldots N_{sub}$ (*K* is the number of all users, N_{sub} is the number of all useable sub-carriers) is assumed to be known in the BS for sub-carrier allocation. The total perceived channel gain P_{total} for all users is considered in order to provide good sub-channels for all users.

The allocation problem then can now be formulated as:

$$\text{Maximize } P_{total} = \sum_{k}^{K} \sum_{n}^{N_{sub}} c_{k,n} |h_{k,n}|^2 \tag{1}$$

Subject to

$$c_{k,n} = \begin{cases} 1, & subcarrier\ n\ is\ assigned\ to\ user\ k, \\ 0, & otherwise, \end{cases} \tag{2}$$

$$\sum_{k} c_{k,n} = 1 \tag{3}$$

$$\sum_{n} c_{k,n} = S \tag{4}$$

where $c_{k,n}$ is the allocation mapping matrix element for user *k* and sub-carrier *n*. N_{sub} is the number of useable sub-carriers and *K* is the number of users (and the number of sub-channels as well in this case). *S* is the number of sub-carriers per user. In this paper (the case of equal numbers of sub-carriers per sub-channel), $S = N_{sub} / K$.

The maximum total perceived channel power is indicated by P_{total}^{opt}.

B. Optimal and sub-optimal solutions

1) OCSA-Extended Hungarian Method

An optimal solution to the above assignment problem can be obtained by an extended Hungarian Method. In order to realize the optimal channel sub-carrier allocation (OCSA) by extending the Hungarian Method, the practical problem has to be reformulated. The manipulation of the new matrix $C \in R^{N_{sub} \times N_{sub}}$ is considered [16]. The relative gain matrix *H* should be changed from size of $K \times N_{sub}$ to $\tilde{K} \times N_{sub}$ where each user's entry is duplicated for *S* times (*S* is the number of sub-carriers allocated per user), i.e. $\tilde{K} = S \times K$. Consequently, the size of this new gain matrix becomes $N_{sub} \times N_{sub}$ to match the extended Hungarian Method.

The optimization problem is re-formulated as

$$\text{Maximize } P_{total}^{N_{sub} \times N_{sub}} = \sum_{n}^{N_{sub}} \sum_{\tilde{k}=1}^{N_{sub}} c_{\tilde{k},n} |h_{\tilde{k},n}|^2 \tag{5}$$

Subject to

$$c_{\tilde{k},n} = \begin{cases} 1, & subcarrier\ n\ is\ assigned\ to\ user\ k\ at\ its\ subcarrier\ l \\ 0, & otherwise \end{cases} \tag{6}$$

$$\sum_{\tilde{k}} c_{\tilde{k},n} = 1 \tag{7}$$

$$\sum_{n} c_{\tilde{k},n} = 1 \tag{8}$$

where $P_{total}^{N_{sub} \times N_{sub}}$ is the total perceived channel gain for *S* times duplication of *K* users, and $\tilde{k}$ is the user or its duplication index $\tilde{k} = 1,2,\ldots,S,S+1,\ldots,2S,\ldots N_{sub}$ .As the allocation mapping matrix element for this user or its duplication $\tilde{k}$ and sub-carrier *n*, $c_{\tilde{k},n}$, is determined, it can be easily returned to $c_{k,n}$ and P_{total}^{opt} can be obtained.

As is well known, the Hungarian Method [9] aims to minimize the cost for the assignment problem. The reciprocal of the channel gain can be used to meet this method exactly, but it might reasonably be expected that the optimal allocation cannot be achieved. Hence it is necessary to invert the method so that the maximum perceived channel gain can be determined.

The sub-carrier allocation by the Hungarian Method is referred to [9] and [16] in which the assignment problem is the inverse (5) to achieve minimum cost for assignment of all the jobs among the individuals such as each individual exactly one job and each job done by exactly one person. [16] uses a cost matrix of *R* by *R* (*R* is number of both individuals and jobs), then searches for the minimum value of the rows and columns through subtracting each element in the rows and columns by the minimum value and minimum lines drawn over all zeros in the processed cost matrix.

In the case of this paper, 'job' can be simulated as 'sub-carrier allocation' and 'individual' as 'user'. The inversion is a modification of the method mentioned above as following basic processing rules based on a reformulated channel gain matrix instead of cost matrix to achieve the allocation mapping matrix [9][16]:

1. Always find maximum element in each row and column of channel gain matrix instead of minimum element in cost matrix;

2. Always make locations with maximum element recorded (such as locating zeros to differentiate other elements).

This is similar to the Hungarian Method as a kind of exhaustive search which makes use of computation to check all pairs of 'job' and 'individual' one by one and compare the total gain for the maximum value. When the size of the channel gain matrix increases (due to more users and/or sub-carriers) the computational time and complexity becomes extremely high.

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Thus, in this paper, the optimal solution is shown as a reference upper bound of the system performance and is NOT suggested for use in practice.

2) MGSS-Sub-optimal solution

Due to the unfeasible complexity of OCSA, a lower complexity sub-optimal solution must be considered. Thus, the algorithm previously proposed in [10] is extended to attempt to achieve maximum perceived channel gain. It is named Maximum Gain Sort-Swap (MGSS) and sorts sub-carrier pairs by metric of total perceived channel gain and swaps sub-carrier allocations between users to exploit the maximum power. The iteration is applied to make the most of the sort-swap process to achieve a near optimal solution. Also, the iterative swap process is used again to improve the DSA algorithm in section III.D. Note that the channel gain matrix which is applied in this solution is H with original and decreased (relative to OCSA) size of $K \times N_{sub}$ (the allocation problem is formulated the same as (1)) and the process in [10] is inverted to give rules:

1. The elements of initial channel gain matrix and processed gain matrix are always sorted in descending order (from highest to lowest).

2. The sub-carriers replacement occurs when the sum of minimum values of all gain increase factor per user [10] is negative.

Due to reasonable and low complexity initial process, the iteration of sort and swap decreases, consequently, resulting in the lower complexity and better practicability relative to OCSA.

C. The DSA algorithm

In this section, an algorithmic definition of the proposed (SISO) DSA scheme is provided. The allocation problem is formulated the same as (1)-(4) and in this solution the channel gain matrix follows the original size. This algorithm is defined here for the SISO case. The various MIMO schemes are discussed later (and detailed in Appendix I and II).

In the following, P_k represents the average received power for user *k*, *K* is the total number of users, N is an M' by N_{Sub} matrix where each row is a vector containing the indices of the useable sub-carriers for the corresponding spatial sub-channel (i.e. the channel that exists between any one transmit element-receive element pair in a MIMO system. The possibility of multiple spatial sub-channels is accommodated in the algorithm at this stage (when considering the SISO case) in order to make the algorithmic definition generic to both SISO and MIMO cases). (i.e. $N_{m'} = \{1,2,3...N_{Sub}\}$), where N_{Sub} is the total number of useable sub-carriers). $m' = \{1...M'\}$ where M' is the effective number of spatial sub-channels considered by the allocation algorithm. $h_{k,n,m'}$ is the channel response for user *k* ($|h_{k,n,m'}|$ manifests the relative channel gain in amplitude), sub-carrier *n* and channel m'. $C_{k,s,m'}$ is a matrix to record the location of allocated sub-carriers for user *k* and sub-carrier (within the sub-channel) *s*. $0_{m',N_{Sub}}$ is a matrix of zeros of size m' by N_{Sub}.

I. Initialization

Set $P_k = 0$ for all users $k = 1...K$

Set $C_{k,s,m'} = 0$ for all users $k = 1...K$ and spatial sub-channels $m' = \{1,2..M'\}$

Set $s = 1$

II. Main process

While $N \neq 0_{m',N_{Sub}}$

{(a) Make a short list according to the users that have less power[1]. Find user k satisfying

$P_k \le P_i$ for all *i*, $1 \le i \le k$

(b) For the user *k* got in (a), Find sub-carrier *n* satisfying

$|h_{k,n,m'}| \ge |h_{k,j,m'}|$ for all $j \in N$

(c) Update P_k, N and $C_{k,s,m'}$ with the *n* from (b) according to

$$P_k = P_k + \sum_{m=1}^{M} |h_{k,n,m'}|^2$$

$$N_{m',n} = N_{m',n} - n$$

$$C_{k,s,m'} = n$$

$$s = s + 1$$

(d) go to the next user in the short list got in (a) until all users are allocated another sub-carrier.

Thus, the algorithm operates by ranking users in order of current allocated (mean) channel gain from lowest to highest. Subsequently, additional sub-carriers are allocated to users in rank order allowing those with the lowest allocated gain to have the next 'choice' of sub-carrier. The operations in the algorithm thus largely consist of sorting, comparing and simple arithmetic.

As well as achieving high multiuser diversity gains (as detailed in the simulation results section), this algorithm has merits in comparison to others including a fair allocation of resources to users and inherent compatibility with link adaptation schemes.

The algorithm is considered in this paper for the case of ideal channel state information (CSI). CSI is important both for DSA and for equalisation, but the proposed algorithm is relatively insensitive to channel estimation errors since only the magnitude of the channel gain is used for the metric to allocate sub-carriers [15].

D. DSA-Swap:Improved 'DSA' algorithm

One possible method to improve the DSA algorithm mentioned in section III.C is to subsequently apply the sort-swap method. The solution of the 'DSA' algorithm is used as an initial solution and subsequently the swap iteration process defined in section III.B. 2) can be applied to try to further improve the allocation. The simulation results show that a better performance can be achieved than the initial DSA algorithm solution and that this method can even reach the

[1] For the first iteration (when no sub-carriers have been allocated and hence all users have equal power) the list may be entirely arbitrary.

performance of the optimal solution after enough (about 3~5 times) iterations.

However, it can be shown in section III.E that the initial DSA algorithm is a near optimal solution for achieving maximum perceived channel gain. In the following sections, this algorithm is called directly 'DSA algorithm' and extended to MIMO-OFDMA.

E. Comparison of Algorithm Performance

The algorithms described above can be compared via software simulation. In this section, the channel model 'E' (defined in section VI) is used.

In order to simplify the (1) and normalize power per user and per sub-carrier:

$$P_{norm}(dB) = 10\log_{10}\left(\frac{1}{16}\times\frac{1}{48}\times\sum_{user=1}^{16}\sum_{sub=1}^{48}\left|h_{user,sub}\right|^2\right)(dB) \quad (9)$$

where $\left|h_{user,sub}\right|^2$ is the channel gain of a certain user for the allocated sub-carrier (1~48 is the allocated sub-carrier index per user, not the index in the real 768 sub-carrier sequence) in a certain simulation time.

The total perceived channel gains offered for 16 users by different algorithms are compared. This has been done for 2000 independent identically distributed (iid) quasi-static random channel instances.

CCDFs of channel gain achieved by DSA and 'conventional' OFDMA (pseudo-random sub-carrier allocation) are compared in Fig. 3. It can be seen that DSA outperforms the random allocation strategy by up to 6dB and results in significantly lower variation around the mean.

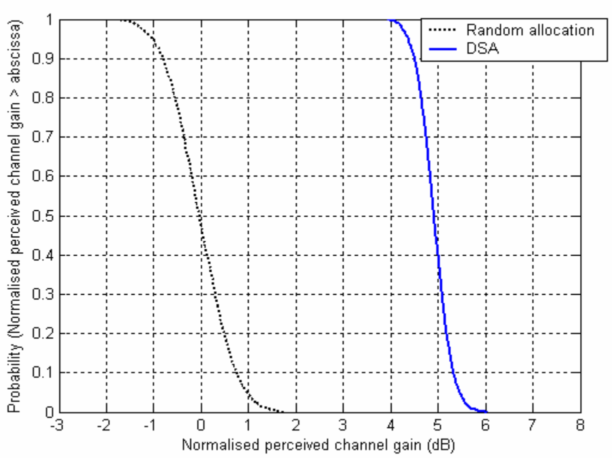

Fig.3. Comparison of CCDF of total channel gain with conventional OFDMA and DSA solutions

These effects can be justified by considering the example of an instantaneous wideband channel response in the frequency domain for a single user and the corresponding sub-carrier allocation achieved by DSA as shown in Fig. 4. This serves to illustrate the multi-user diversity benefit which the DSA algorithm is able to achieve. It can be seen that the sub-carriers allocated in this instance have consistently high gain (all are higher than the mean for the channel over all sub-carriers) and a much flatter response than the actual channel. These factors lead to the change in channel statistics illustrated in Fig. 3 and the BER performance gains demonstrated and discussed in section VII. It can be intuitively seen how an alternative user perceiving an uncorrelated channel response could derive similar benefit from a different set of sub-carriers. It can also be seen that a pseudo random (or clustered) sub-carrier allocation would be unable to achieve this multi-user diversity gain.

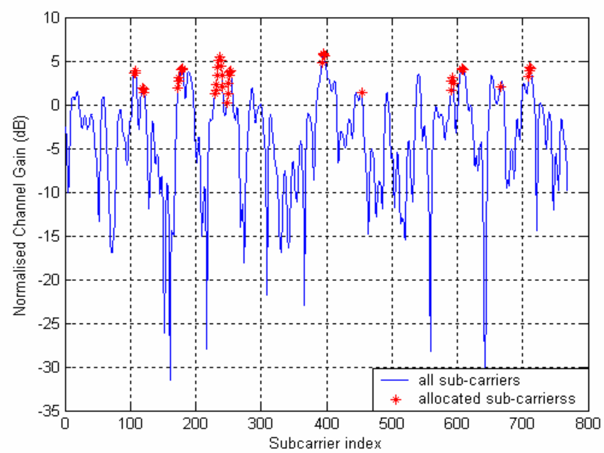

Fig. 4. Example Channel Response and Sub-carrier Allocation

The resulting power gains as a function of iteration number is illustrated for one typical channel instance in Fig. 5. Both MGSS and DSA-Swap tend very close to the optimum solution (achieved by OCSA) after 3-5 iterations. The result of DSA-swap is slightly better than that of MGSS. Care should be taken to note the scale on the power gain axis since, whilst DSA and OCSA look far apart on this graph, DSA actually achieves approximately 97.54% of the power gain of OCSA. Hence, the DSA algorithm (without swapping) can be considered a low complexity, near optimal solution. MGSS is also worthy of interest because it also offers a low complexity, near optimal sub-carrier allocation. DSA-swap is of less interest since it offers minimal improvements over DSA and MGSS in return for increased complexity (it is essentially a concatenation of the two).

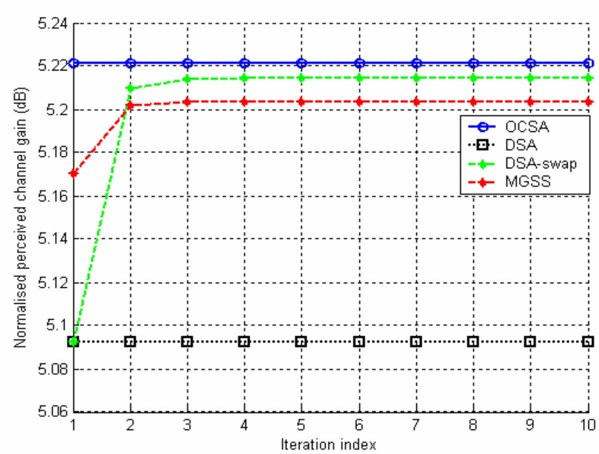

Fig. 5. Comparison of all solutions:OCSA, DSA, DSA-swap, and MGSS

Fig. 6 presents the complementary cumulative distribution

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function (CCDF) of all 2000 random channels for DSA, DSA-Swap and MGSS (OCSA cannot be shown because of high complexity in simulation). This shows that the similar performance of these algorithms is consistent across the entire statistical sample.

Having established that all the above algorithms achieve near optimal performance, the DSA algorithm is identified as the

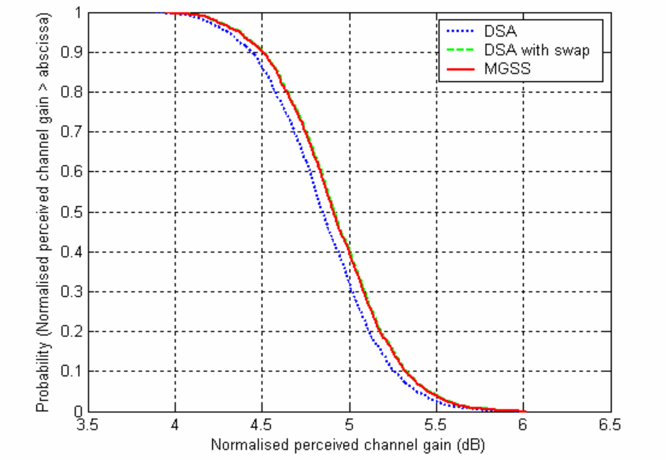

Fig.6. Comparison of CCDF of total channel gain for all sub-optimal solutions

algorithm of primary interest due to one further feature – it is readily extendable to the MIMO case and is particularly amenable to enhancements to combat the debilitating effects of correlation in the radio channel as discussed in the following sections.

IV. The DSA Schemes in MIMO-OFDMA

3 'schemes' to extend the DSA algorithm to MISO (2Tx 1Rx) and MIMO (2Tx 2Rx and 4Tx 4Rx) systems are initially proposed. These schemes are designed to transmit a certain user's symbols on the sub-carriers which are allocated by the metric from perceived spatial sub-channel gain. There is no more than one user per sub-carrier per spatial sub-channel. Scheme 1 considers sub-carrier allocation for each spatial sub-channel separately; schemes 2 and 3 allocate sub-carriers jointly for all spatial sub-channels. These 3 schemes are extensions of the core algorithm detailed in section III.C, and are detailed in Appendix I.

A. *Scheme 1*

The DSA algorithm is separately applied for each (spatial) sub-channel to determine the sub-carrier allocation. This is a straightforward extension of the DSA algorithm which takes no account of correlation and no consideration of one spatial sub-channel relative to the other spatial sub-channels.

B. *Scheme 2*

This scheme attempts to exploit correlation (primarily to the benefit of STBC systems) by choosing the same sub-carrier allocations for all spatial sub-channels. This scheme allocates each sub-carrier on the basis of the maximum channel gain of all the spatial sub-channels for that sub-carrier and user.

C. *Scheme 3*

This is an alternative to scheme 2 which allocates each sub-carrier on the basis of the average channel gain of all the spatial sub-channels for that sub-carrier and user.

D. *Comparison of schemes 1, 2 and 3*

Since scheme 1 considers DSA of each spatial sub-channel separately (using different sets of sub-carriers in spatial sub-channels), performance in each spatial sub-channel can achieve the same DSA gain as in a SISO system with the DSA algorithm. If spatial channels are un-correlated, spatial diversity gain is as high as it can be (relative to the equivalent MIMO system in a correlated channel). An increased SNR gain can be achieved because of receiver diversity and more freedom for the base station to allocate available sub-carriers.

Scheme 2 and scheme 3 enforce the same sub-carrier allocation for all spatial sub-channels. The effect of this allocation is similar to that of adding extra multiple components with the same allocated sub-carriers by 'DSA' algorithm with metric of maximum (scheme 2) or average (scheme 3) channel gain over all spatial sub-channels. The system can enjoy both spatial diversity gain and DSA gain as well. This provides for a simpler algorithm and implementation without additional hardware irrespective of the DSA mechanism or channel equalization at the receivers. But it might reasonably be expected that the DSA gain of each spatial sub-channel will be reduced.

V. The DSA Schemes to Combat Channel Correlation

As is well known, the spatial correlation properties of the radio channels are a key to MIMO system capacity. Two adaptive multi-user sub-carrier allocation algorithms extended from the schemes described above are applied to combat the debilitating effects of correlation on a MIMO-OFDMA system. For each user, the sub-carrier allocation is performed in a fashion which reduces correlation whilst still seeking a near-maximal allocation of channel energy. The details of the processes of these two schemes are described in Appendix II.

A. *Scheme 4*

As with scheme 1, the dynamic sub-carrier allocation is applied independently across spatial sub-channels. Additionally, a check is performed to identify cases in which the same sub-carrier has already been allocated to the same user in a previously considered spatial sub-channel. If this occurs, that sub-carrier will be replaced by the next best sub-carrier (and the previous allocation check repeated for that sub-carrier).

There is an example for scheme 4 (Fig. 7). Provided there are two spatial sub-channels A (first row in the figure) and B (second row in the figure), the sub-carrier allocation of a certain user (sub-channel) has been decided by DSA algorithm based on channel gain of spatial sub-channel A. Currently, sub-carriers should be allocated for the same user under spatial sub-channel B. The best sub-carrier through ranking by metric of channel gain is '15' (the number of a certain sub-carrier). However, it can be found that '15' has been already used for the same user in different spatial sub-channel. The '15' has been abandoned and the next best sub-carrier '17' is chosen by the rules of scheme 4.

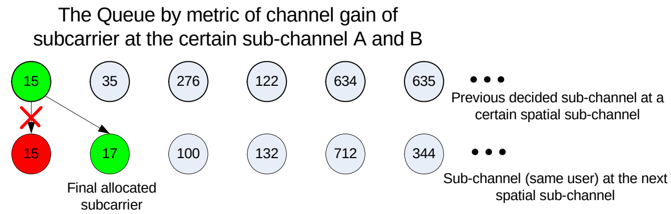

Fig. 7. An example for scheme 4

B. Scheme 5

In this scheme, not only is the allocation of the same sub-carrier to different spatial sub-channels prevented but the allocation of near adjacent sub-carriers is prevented also. The number of near adjacent sub-carriers avoided in this process is denoted *q*. It is noteworthy that *q* may be assumed to take a value of 1 in scheme 4 and 0 in scheme 1 in which case schemes 1 and 4 may be viewed as sub-sets of scheme 5.

Similar to scheme 4, there is an example for scheme 5 (Fig. 8). Under sub-channel B, not only '15' is abandoned, the '17' and '10' which fall within range of '15'+/-10 (here *q* is equal to 10) are not allowed for this user. The rule of scheme 5 is forcing the use of sub-carrier 132 even though it is 3 steps further down the ranking list.

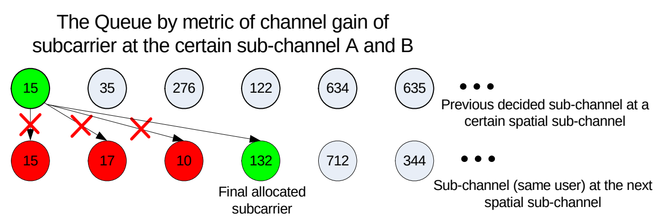

Fig. 8. An example for scheme 5

C. Comparison between these schemes

Since the channel is frequency-continuous, assigning symbols to the adjacent sub-carrier locations of an initially chosen sub-carrier location should perform similarly (the exception being the case where the coherence bandwidth is not significantly greater than the sub-carrier spacing). If the channels are correlated, the adjacent sub-carrier locations in other spatial sub-channels will also be correlated to some extent. Scheme 4 only considers the spatial correlation if the same sub-carriers are chosen. Scheme 5 will reduce spatial correlation further but at the expense of some reduction in the DSA gain.

VI. Simulation Environment And Parameters

The Performance of the DSA algorithm is evaluated by simulation. The simulation considers mainly QPSK modulated, rate-1/2 convolutionally coded (CSI-soft Viterbi decoded), COFDM operating with a bandwidth of 100MHz as a candidate 4G physical layer. In the SISO-OFDMA case, the simulations at different modulation modes with different convolutional coding rates (Table I) are performed as well for comparison purposes. Note that in Table I, the values in brackets specify the parameters for SM with 2 transmitters, and the values not in brackets specify the parameters for the cases of SISO and STBC with 2 transmitters.

TABLE I
Modulation Parameters in OFDMA

Modulation	QPSK	QPSK	16QAM	16QAM	64-QAM
Coding Rate	1/2	3/4	1/2	3/4	3/4
Data bits per sub-channel	48 (96)	72 (144)	96 (192)	144 (288)	216 (432)
Data bits per OFDM symbol(all 16channels)	768 (1536)	1152 (2304)	1536 (3702)	2304 (4616)	3,456 (6912)
Total Bit Rate [Mbit/s]	64 (128)	96 (192)	128 (256)	192 (384)	288 (576)
Coded bits per sub-channel	96 (192)	96 (192)	192 (384)	192 (384)	288 (576)

A multipath channel with excess delay of 1600ns with each path suffering from independent Rayleigh fading was used in [17, 18] for the performance evaluation of similar 4G candidate physical layer proposals. On this basis, a similar channel model – referred to as channel 'E' – is used here, which is specified by ETSI BRAN [19]. Channel model 'E' corresponds to a pico-cell type outdoor environment with NLOS conditions and large delay spread [20]. The rms delay spread is 250ns and the excess delay is 1760ns with tap spacing of 10ns.

An addition channel scenario is also considered to investigate the performance in an environment with higher multipath delays. A channel model corresponding to a vehicular micro-cell environment that was employed in the development of 2G and 3G [21,22] – referred to here as channel model 'V' – is employed. This channel model has an rms delay spread of 370ns and an excess delay of 2690ns with the same tap spacing as channel model E.

TABLE II
Channel Model

Name	'E'	'V'
RMS delay spread (ns)	250	370
Excess delay (ns)	1760	2690

The use of both Space-Time Codes (STBC) and Spatial Multiplexing (SM) is considered. 2000 iid quasi-static random channel samples are used in each simulation and the sub-carrier allocation is updated via the appropriate DSA scheme for each such sample.

It is assumed that the DSA algorithm is implemented by the BS and that the BS has perfect knowledge of the channel gain matrix and uses this to determine sub-carrier allocation. Furthermore, it is assumed that the MSs have perfect knowledge of the channel transfer function for those sub-carriers allocated to them and that this is used for equalisation and decoding purposes.

For OFDMA and DSA algorithms, 16 users are considered and there are 768 usable sub-carriers in all. Simulation parameters are summarized in Tables I and II.

A. In un-correlated MIMO channels

All three schemes for MIMO are first simulated in un-correlated channels with results presented in section V.

B. Simulations in correlated MIMO channels.

The simulation is further extended to consider correlated MIMO channels. The derived models are based on [23] which includes the partial correlation between the paths in the channel. 2 antennas at the Base Station (BS), and 2 antennas at the Mobile Station (MS) are considered. The MS is simulated in an urban environment surrounded by numerous or few local scatters which results in the lower or higher correlation between two antennas. BS antennas are located on the rooftop level of the surrounding buildings, which follows a Laplacian function in a typical urban environment. The correlation scenarios considered can be seen in Table III. The spatial correlation matrix of the MIMO radio channel R_{MIMO} is the Kronecker product of the spatial correlation matrix (R_{BS} , R_{MS}) at the BS and the MS [24,25]. A selection of results for different correlation scenarios are shown for STBC and SM.

TABLE III
CORRELATION SCENARIOS

Correlation Modes	MIMO	
	R_{BS}	R_{MS}
HL	0.91	0.30
HH	0.91	0.91
'Full'	0.99	0.99

VII. SIMULATION RESULTS

In order to evaluate the proposed algorithm and schemes and compare the performances in different cases, the following simulation results are presented.

A. The DSA algorithm in SISO-OFDMA

In section III, it was confirmed that the initial DSA algorithm [3] achieves a near optimal sub-carrier allocation. So in the following, this algorithm is considered in all cases and named directly 'DSA algorithm'. In this section, the BER performance is presented for SISO OFDMA with and without the DSA algorithm.

Fig. 9 compares the mean performance of all 16 users in channel 'E' when operating at 64Mbit/s (1/2-rate QPSK) both with and without DSA. It can be seen that very substantial gains (~11dB at 10^{-4} BER) can be achieved by DSA. Whilst this may seem surprisingly high, it must be considered in the context of the large amount of multi-user diversity gain demonstrated in section III.

Fig. 10 compares the performance of a sample of different users in the system employing DSA (again for channel 'E' and the 64Mbits/s data rate, the average performance without DSA is shown again for reference). It can be seen that the performance of the users is extremely consistent, there is minimal variation in performance (as a function of received SNR) between users. Only a sample of the 16 users is shown for clarity but the sample is a fair representation of the full set of users – performance gains are consistent across all users.

It should be noted that user performance is 'equal' on the basis of a comparison of BER against received SNR. Thus, the gain provided to each user by DSA is equal. This does not imply however that the performance of all users is truly equal (this is not likely in a real world environment where fast fading, shadowing and free space attenuation will result in spatially diverse users seeing substantially different radio propagation conditions) nor that the DSA algorithm acts to compensate disadvantaged users.

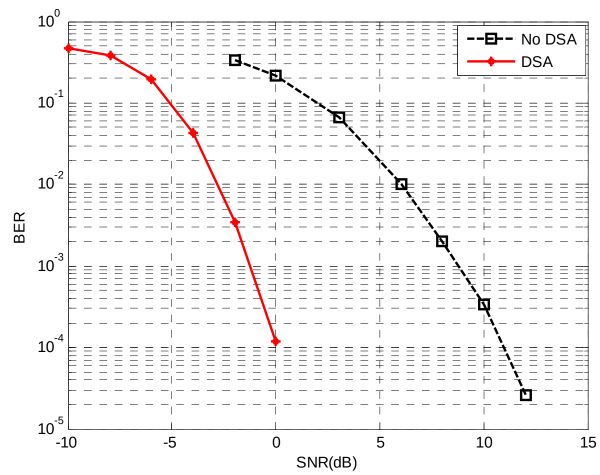

Fig. 9. Average performance of all users in channel 'E' (64Mbit/s)

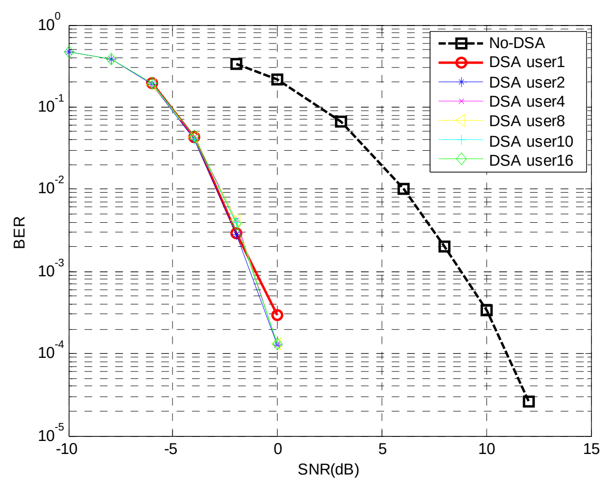

Fig. 10. Performance of a sample of different users in channel 'E' (64Mbits/s)

Fig. 11 shows the performance gain achieved by DSA in channel 'V', again for the 64Mbit/s data rate. Again, substantial benefits are evident (~7dB at 10^{-4} BER).

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Fig. 12 shows the performance gain achieved by DSA in channel ‘E’ for the case of the 288 Mbit/s data rate. Whilst the higher modulation order and coding rate naturally results in increased SNR requirements, the gains achieved by the DSA algorithm are actually higher (~14dB at 10^{-4} BER). This can be attributed to the fact that the heavily punctured 3/4-rate code is less able to average errors in the highly frequency selective channel perceived by the receiver in the case where DSA is not employed. When DSA is employed, as described above, it has the effect of reducing the variation of the perceived channel in the frequency domain, thereby reducing the requirement for the code to average out fading effects. This implies that DSA has the benefit of facilitating the use of higher rate error correcting codes. It can be confirmed again in Fig. 13 which lists and compares all modulation orders and coding rates. As an example, there is 6dB loss from QPSK 1/2 to QPSK 3/4 in the case without DSA, but only 3dB loss in the case with DSA.

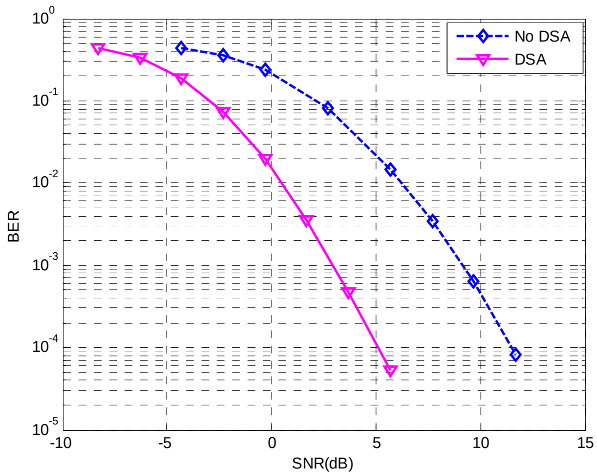

Fig. 11. Average performance of all users in channel ‘V’ (64Mbit/s)

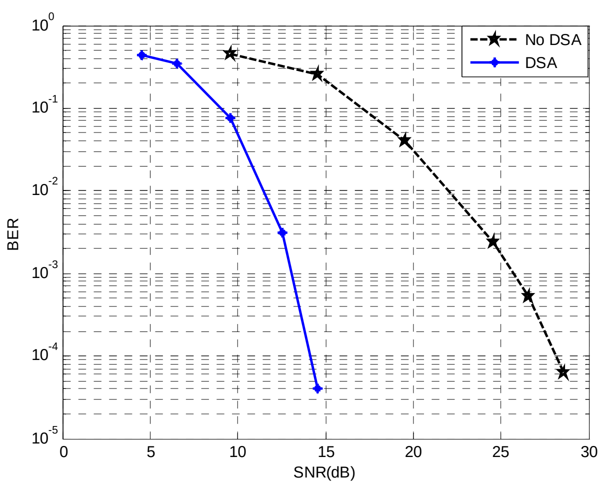

Fig. 12. Average performance of all users in channel ‘E’ (288Mbit/s)

B. *The extended DSA algorithms in a MIMO-OFDMA system*

A selection of results is shown as Figs. 14-26. Results cover MIMO OFDMA systems both with and without DSA and with either STBC or SM and for various correlation scenarios. Although the scheme 2 has been simulated, results are not shown for all cases. This is since performance of this scheme is universally similar to, but never better than, scheme 3 (as illustrated in Fig. 15).

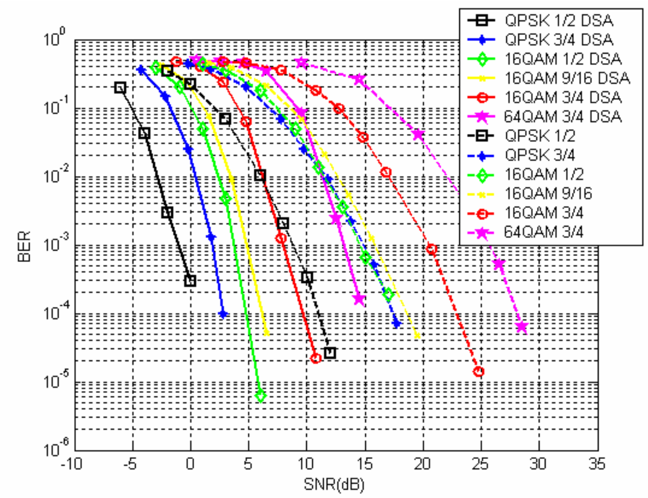

Fig. 13. Performance of a sample of different modulation orders and coding rates in channel ‘E’

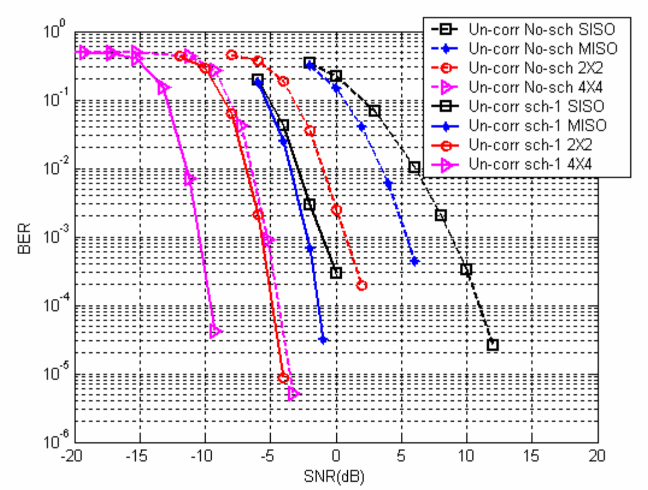

Fig. 14. Un-correlated SISO and STBC with scheme 1

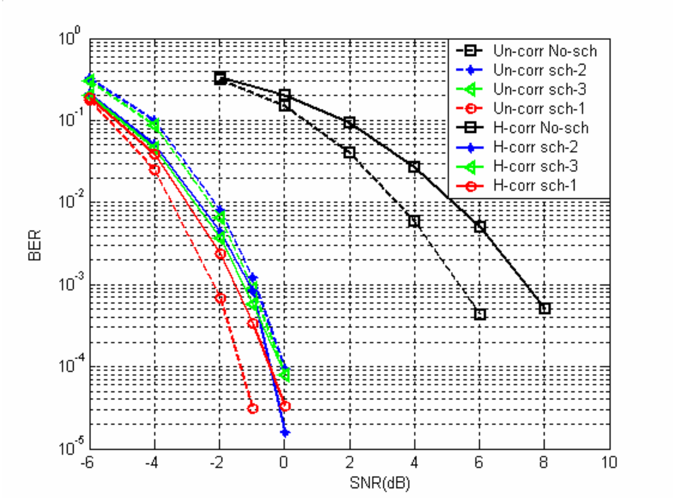

Fig. 15. STBC MISO with Correlation Mode H

Results presented for scheme 5 are for the case where *q=10*. This value was chosen on the basis of some crude optimization via a trial and error approach and is not necessarily optimal.

1) BER performance In un-correlated MIMO channels

All schemes have been simulated in un-correlated channels and the relevant results are presented for reference in many of the following graphs. Also shown for reference, where relevant, is the performance of the equivalent system without any DSA applied – 'No-Sch.'

DSA achieves a gain in the SISO system of more than 10dB at 10^{-4} BER (Fig. 14). Scheme 1 performs similarly well in uncorrelated channels (Fig. 14), achieving approximately 8dB gain for STBC MISO, around 6.5dB gain for STBC MIMO (2Tx 2Rx) and 5dB for STBC MIMO (4Tx 4Rx). (For simplicity, all following results for STBC MIMO is 2Tx 2Rx). 8dB gain is also achieved for SM MIMO (Fig. 21).

It can be seen that for the case of STBC, schemes 1, 4 and 5 achieve the best BER performance and the (sub-carrier allocation) gain closest to that of the SISO system (~ 7-8dB according to the uncorrelated curves in Fig. 16, 17 and 18). The difference in performance between schemes 1, 4 and 5 is negligibly small. Schemes 1, 4 and 5 always outperform schemes 2 and 3.

For the case of SM it can be seen from Figs. 21, 22 and 23 that scheme 1 narrowly outperforms scheme 4 which in turn narrowly outperforms scheme 5. This might be intuitively expected since schemes 4 and 5 sacrifice to greater degrees the selection of the best available sub-carrier in preference for avoiding allocation of the same or nearby sub-carriers on different spatial sub-channels. For an uncorrelated channel, such sacrifice achieves no benefit. It can also be seen in Fig. 21 that scheme 3 offers the worst performance.

2) BER performance in correlated MIMO channels

It is shown that for correlated channels, in comparison to the results for un-correlated MIMO channels, Scheme 1 is somewhat impaired by the correlation with the degree of impairment increasing with increasing correlation (Fig. 16, 17 and 18 and summarized in 19 for STBC; Fig. 21, 22 and 23 and summarized in 24 for SM). It can be seen by comparison between Fig. 19 and 24 that (as might reasonably be expected) the impairment due to correlation is much more severe for the case of SM than for STBC.

In comparison, it can be seen (Fig. 16, 17 and 18 and summarized in 20) that scheme 3 is actually able to derive some benefit from the correlation when STBC (but not SM) is used. As can be seen from Fig. 18 however, even for the 'full' correlation scenario, scheme 3 only just outperforms schemes 1 and 4 at the upper limit of correlation and still under-performs scheme 5.

As can be seen in Fig. 21, 22 and 23, scheme 4 and scheme 5 mitigate the effects of correlation on SM MIMO by different degrees. For HL, HH and 'Full' correlation scenarios, Scheme 5 achieves the best BER performance. The degradation in performance of scheme 5 under increasing correlation can be seen in Fig. 26 and is relatively graceful. A loss of approximately 1.8dB is evident between the un-correlated and 'full' correlated cases at a BER of 10^{-4}. Scheme 4 degrades somewhat more severely under increasing correlation, with a loss of 3.2dB (Fig. 25) at the same BER. As discussed, scheme 1 degrades much more severely than schemes 4 and 5 and is unable to achieve a BER of 10^{-4} in the 'full' correlation scenario due to the presence of a distinct error floor (Fig. 24).

Fig. 23 summarises the relative performances of schemes 1,4 and 5 for SM at the extremes of correlation, showing the slight advantages of scheme 1 over scheme 4 and scheme 4 over scheme 5 in uncorrelated channels and the significant advantage of scheme 5 over scheme 4 and a further large advantage of scheme 4 over scheme 1 in the 'full' correlated channel.

In addition, with correlation increasing, performance in the absence of any DSA scheme becomes very poor. The benefits of implementing a DSA algorithm consequently become greater as correlation increases. The benefits of schemes 4 and 5 over the case without DSA increases from about 8dB in the uncorrelated channel to more than 11dB in the 'full' correlated channel for STBC and from 14dB in the uncorrelated channel to more than 20dB in the HH correlated channel for SM.

3) Performance comparison betweens STBC and SM

It is shown that systems employing STBC can survive the effects of correlation better than those employing SM. Scheme 4 and scheme 5 can get slightly better BER performance than scheme 1 for STBC. In highly correlated channels, such as "Full" correlation mode, the performance of scheme 3 is beyond schemes 1 and 4 and very close to scheme 5.

However an SM system is very sensitive to correlation effects. As the correlation among channels increases to 'Full,' the BER performance with scheme 1 comes to an error floor. However, as discussed above, with scheme 5, at a BER of 10^{-4} there is only a small degradation due to correlation. This result clearly demonstrates the capability of a well designed sub-carrier allocation algorithm to combat the debilitating effects of channel correlation on MIMO systems.

4) Comparison in performances of scheme 5 with various q

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The variable q in scheme 5 is investigated. Examples are shown as Fig. 27-28. Scheme 4 is also compared as a special case for $q=1$ and scheme 1 as a special case for $q=0$. Furthermore, the *case* $q=0$ can be considered the optimal value in un-correlated MIMO-SM systems. Fig. 27 and Fig.28 show the comparison of SNR requirement for the target BER of 10^{-4} as a function of q for both of STBC and SM cases. It can be seen that in the case of channel 'E', when q increases, less SNR is needed to achieve BER of 10^{-4} until q reaches a value of 10. Values of q larger than 10 require more SNR, although not as much as the cases of $q=0$ and 1. This is because when q increases, the effect of correlation and loss of spatial diversity can be reduced whilst the selection of the 'best' sub-carriers (in terms of channel gain) are sacrificed. As a result, there is a tradeoff between mitigating correlation and achieving DSA gain. For different channel model or correlation modes, the optimal value may change.

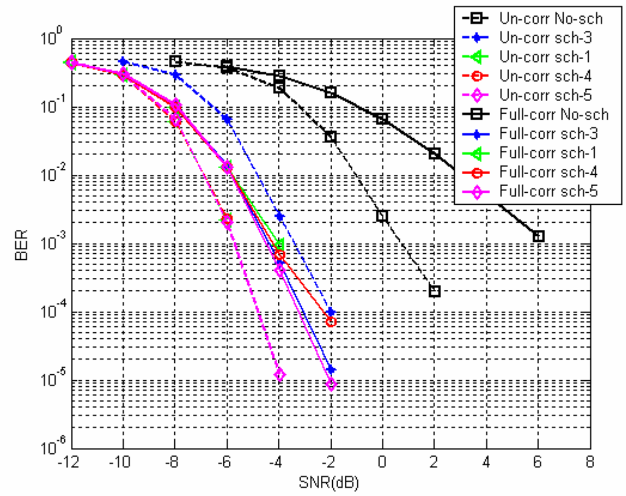

Fig. 18. STBC MIMO with correlation mode 'Full'

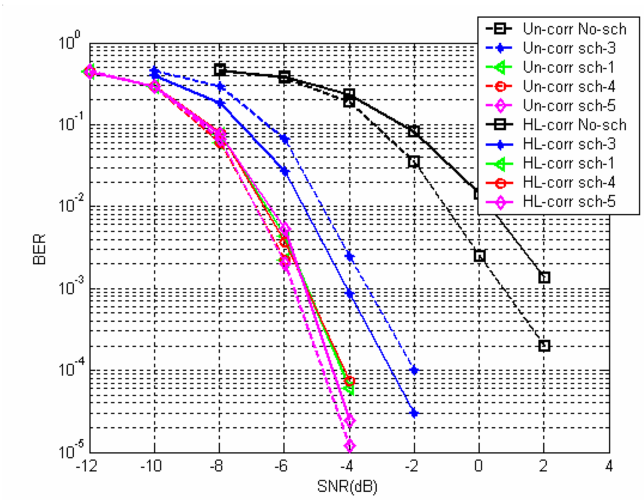

Fig. 16. STBC MIMO with correlation mode HL

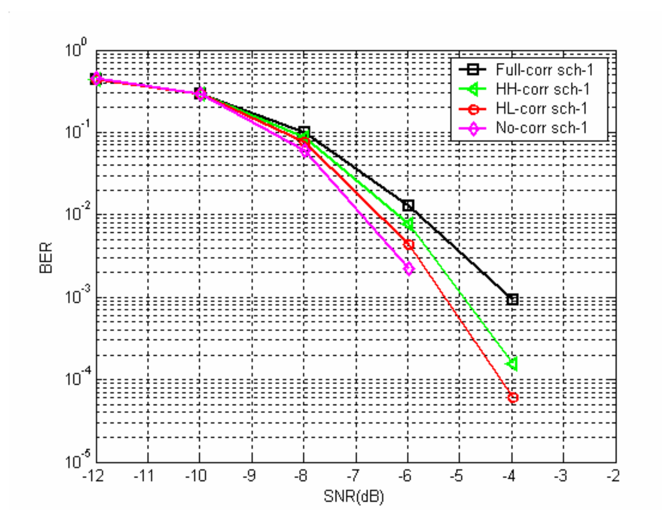

Fig. 19. STBC MIMO scheme 1

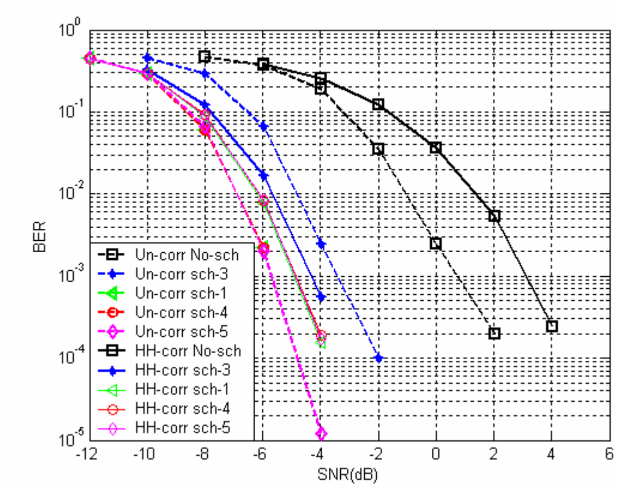

Fig. 17. STBC MIMO with correlation mode HH

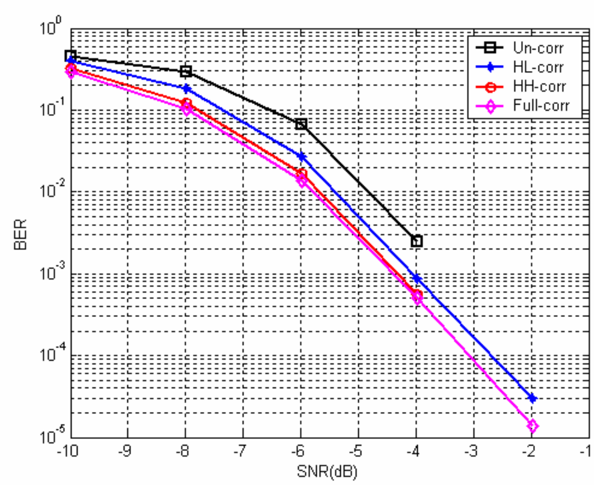

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Fig. 20. STBC MIMO scheme 3

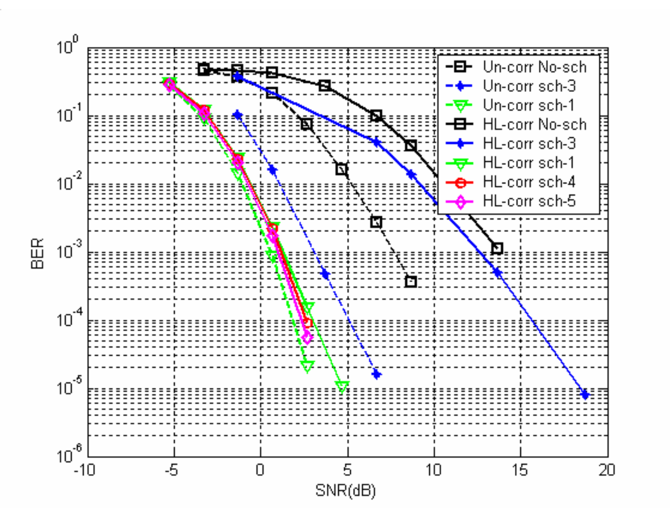

Fig. 21. SM 2X2 with correlation mode HL

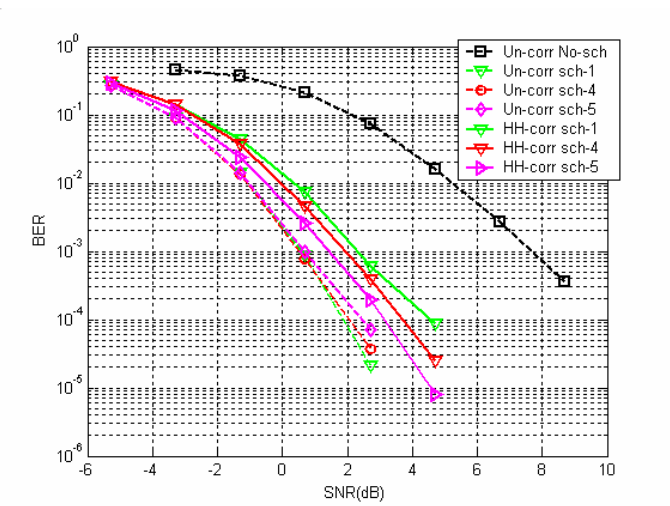

Fig. 22. SM 2X2 with correlation mode HH

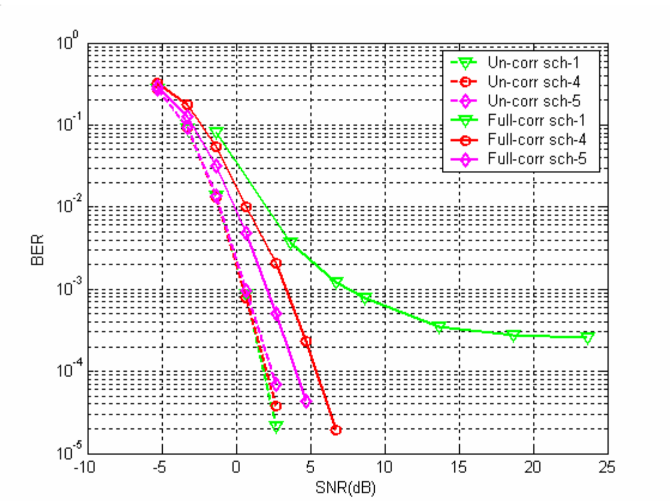

Fig. 23. SM 2X2 with correlation mode 'Full'

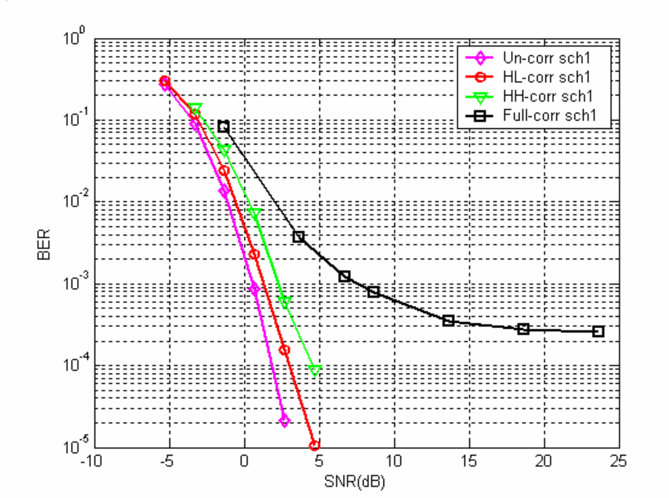

Fig. 24. SM 2X2 with scheme 1

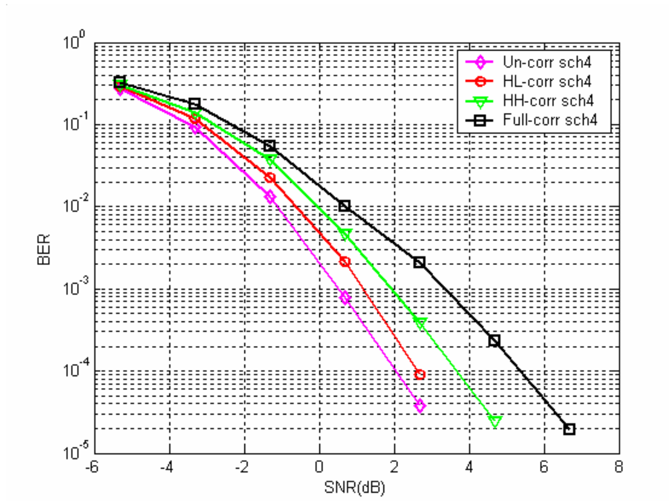

Fig. 25. SM 2X2 scheme 4

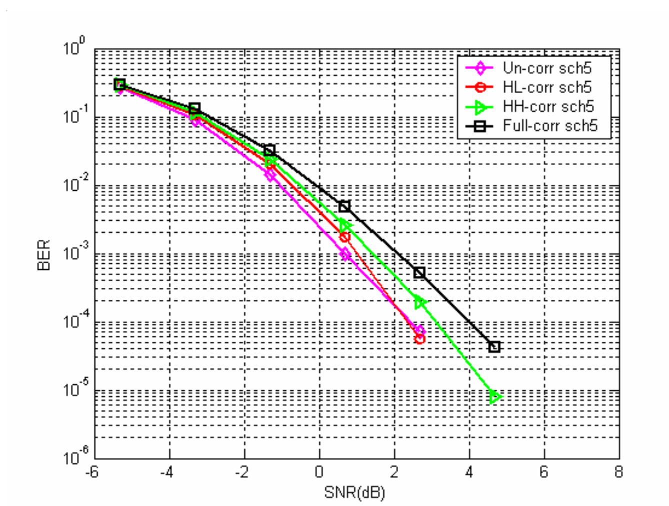

Fig. 26. SM 2X2 scheme 5

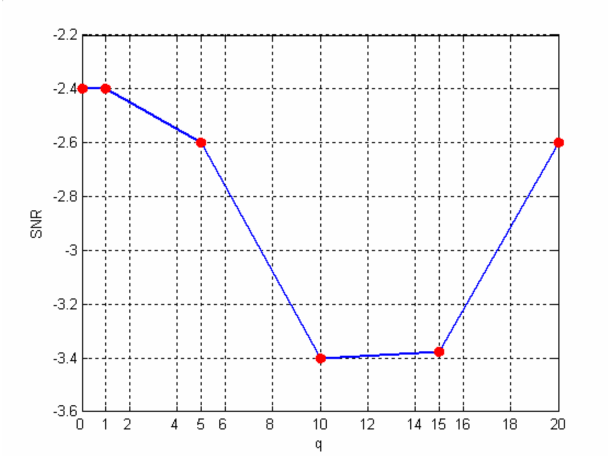

Fig. 27. SNR performance for STBC under various *q* at BER of 10^{-4}

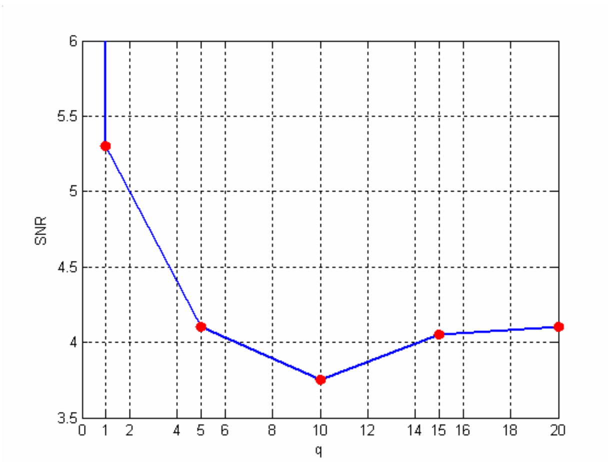

Fig. 28. SNR performance for SM under various *q* at BER of 10^{-4}

VIII. Conclusion

In this paper, an algorithm for Dynamic Sub-Carrier Allocation has been proposed and evaluated in terms of performance in a '4G' mobile broadband WWAN context. The schemes proposed for MIMO systems are the first to consider Dynamic Sub-Carrier Allocation as a means to combat the debilitating effects of spatial correlation in the wireless channel.

For the SISO case, results show that the DSA algorithm is capable of exploiting the flexibility of fine granularity frequency allocation facilitated by OFDMA to derive substantial performance gains from multi-user diversity. Compared with other heuristic algorithms, this DSA algorithm is identified to be a near optimal solution to the sub-carrier allocation problem with relative low complexity. Results show that gains vary between 7 and 14dB depending upon the channels and modulation and coding schemes considered and that the gains are consistent across users, implying that the algorithm has the additional benefit of achieving a very fair distribution of multi-user diversity benefits between users. The results for higher data rates also imply that DSA facilitates the use of higher rate FEC codes.

For the MIMO case, considering both un-correlated and correlated MIMO channels, scheme 5 would appear to be the superior option: achieving near optimal performance in the un-correlated case and providing the greatest degree of robustness to the effects of correlation (less than 2dB degradation from the full correlated to un-correlated case). Scheme 1 and 4 (which can be considered to be special cases of scheme 5) cannot reach the best capability to mitigate the debilitating effects of correlation on MIMO systems (scheme 1 is even exhibits an error floor); Schemes 2 and 3 are less complex and can get benefit from the correlation, but generally perform significantly worse than the other schemes.

Scheme 5 achieves this good performance is spite of the fact that the value of *q* used has not been studied in detail. This is an obvious area for further work and may yield further improvements in performance. Use of an adaptive value for *q* should be considered as a means of getting the best performance across the full range of correlation scenarios. It can be expected that as *q* is changed, the variability of the channel power allocation will be changed. Also variation of perceived channel gain will become stronger when *q* is increased.

The core DSA algorithm and the various MIMO schemes all consists of relatively low complexity operations (loops, sorting and comparison). Given the substantial performance benefits that the algorithm has been shown to offer, an attractive cost-benefit tradeoff might be inferred. However, a detailed, qualitative analysis of implementation complexity remains an obvious subject for further work.

Appendix

I. Nomenclature

Symbol	Applications and Notes
$0_{m',N_{Sub}}$	A matrix of zeros of size m' by N_{Sub}
$c_{k,n}$	Allocation mapping matrix element for user *k* and sub-carrier *n*
$c_{\tilde{k},n}$	Allocation mapping matrix element for a user or its duplication $\tilde{k}$ and sub-carrier *n*
$C_{k,s,m'}$	A matrix to record the location of allocated sub-carriers for user *k* and sub-carrier (within the sub-channel) *s*
k	User
$\tilde{k}$	User or its duplication
K	The number of all users
$\tilde{K}$	The number of *S* times duplication of *K* users, $\tilde{K} = S \times K$
$h_{k,n}$	Channel response for user *k* , sub-carrier *n* for a certain channel
$h_{k,n,m'}$	The channel response for user *k*, sub-carrier *n* and channel m'.
$\lvert h_{k,n} \rvert$	Channel transfer function amplitude
$\lvert h_{k,n,m'} \rvert$	The relative channel gain in amplitude
$\lvert h_{user,sub} \rvert^2$	The channel gain of a certain user for the allocated sub-carrier in a certain simulation time

H	Channel gain matrix, $H=\{\lvert h_{k,n}\rvert^2\}, k=1,2,\ldots,K, n=1,2,\ldots N_{sub}$
L	The alternative subcarrier in scheme 4 and 5
m	A certain spatial sub-channel
m'	Spatial sub-channel index $m'=\{1\ldots M'\}$
M	The true number of spatial sub-channels ($M=T_x\times R_x$)
M'	The effective number of spatial sub-channels considered by the allocation algorithm
n	A certain subcarrier
N_{sub}	The number of all useable sub-carriers
N	An M' by N_{Sub} matrix where each row is a vector containing the indeces of the useable sub-carriers for the corresponding spatial sub-channel. (i.e. $N_{m'}=\{1,2,3\ldots N_{Sub}\}$)
P_{total}	The total perceived channel gain, $P_{total}=\sum_k^K\sum_n^{N_{sub}} c_{k,n}\lvert h_{k,n}\rvert^2$
P_{total}^{opt}	The maximum total perceived channel power
$P_{total}^{N_{sub}\times N_{sub}}$	The total perceived channel gain for L times duplication of K users $P_{total}^{N_{sub}\times N_{sub}}=\sum_n^{N_{sub}}\sum_{\tilde{k}=1}^{N_{sub}} c_{\tilde{k},n}\lvert h_{\tilde{k},n}\rvert^2$
P_{norm}	Normalized power per user and per sub-carrier
P_k	The average received power
q	The number of near adjacent sub-carriers avoided in scheme 5 process. The value of *10* was chosen on the basis of some crude optimization via a trial and error approach and is not necessarily optimal.
R_x	The number of receive antennas
s	Subcarrier index
S	The number of sub-carriers per user, In this paper (the case of equal numbers of sub-carriers per sub-channel), $S=N_{sub}/K$.
T_x	The number of transmit antennas

II. The Algorithms of 'Schemes' 1-3

A. Scheme 1

This scheme can be defined by nesting the core algorithm (section IIIC) within a loop for all spatial sub-channels, i.e.:

For *m = 1* to *M*

{As $m\le T_x$ Perform Core Algorithm;

As $m>T_x$, sub-carrier *n* got in step (b) is checked:

Find $s_1,s_2,\ldots,s_{m-1}$ for $C_{k,s_1,1}=n$, $C_{k,s_2,2}=n$ … $C_{k,s_{m-1},m-1}=n$. If two of them are different and *s* is not equal to any of them, we get the second sub-carrier *n'* satisfying $\lvert h_{k,n,m'}\rvert\ge\lvert h_{k,n',m'}\rvert\ge\lvert h_{k,j,m'}\rvert$ for all $j\in N-\{n\}$ to avoid applying the same sub-carrier to more than two allocations. Then go to (c) with *n'* instead of *n*. The rest is same as Core Algorithm.}

where M is the true number of spatial sub-channels ($M=T_x\times R_x$, T_x is the number of transmit antennas, R_x is the number of receive antennas). In the process, the constraints of $m\le T_x$ and $m>T_x$ ensure that each subcarrier is allocated only T_x times obeying Shannon capacity theorem.

B. Scheme 2

This scheme can be defined by replacing step (b) (the shaded part) of the core algorithm (section IIIC) with this loop process:

{(b)(i) According the *k* got in (a)

Find sub-carrier n_m for all $M=T_x\times R_x$ channels satisfying:

$$\lvert h_{k,n_m,m}\rvert\ge\lvert h_{k,j_m,m}\rvert \qquad \text{for all } j_m\in N$$

(b)(ii) Find the maximum value among the values got in (b)(i).

$$h\_\max_{k,n,m}=\max(\lvert h_{k,n_m,m}\rvert)$$

Then go to (c) with n to update P_k, N and $C_{k,s,m}$. The rest is same as Core algorithm.}

C. Scheme 3

Scheme 3 is an alternation of scheme 2. It can be similarly defined by replacing step (b) (the shaded part) of the core algorithm with:

(b)(i) According the *k* got in (a)

$$h\_ave_{k,\tilde{n}}=\sum_{m=1}^{M}\lvert h_{k,n_m,m}\rvert \qquad \text{for all } \tilde{n}\in N_m$$

(b)(ii) For the user *k* found in (b)(i), find sub-carrier *n* satisfying

$$h\_ave_{k,n}\ge h\_ave_{k,j} \qquad \text{for all } j\in N_m$$

The *n* got from above process is brought into (c).

III. The Algorithms of Scheme 4 and 5

A. Scheme 4

As *m = 1* Perform same as scheme 1

For *m = 2* to *M*

{As $m\le T_x$, go to (*)

As $m>T_x$, sub-carrier *n* got in step (b) is checked:

Find $s_1,s_2,\ldots,s_{m-1}$ for $C_{k,s_1,1}=n$, $C_{k,s_2,2}=n$ … $C_{k,s_{m-1},m-1}=n$. If two of them are different and *s* is not equal to any of them, we get the second sub-carrier *n'* satisfying $\lvert h_{k,n,m'}\rvert\ge\lvert h_{k,n',m'}\rvert\ge\lvert h_{k,j,m'}\rvert$ for all $j\in N-\{n\}$ to avoid applying the same sub-carrier to more than two allocations.

No matter $m\le T_x$ or $m>T_x$, process following:

(*) *L* = *n* (if $m=T_x$) (or *n'* if $m>T_x$ and process above) (*L* is a variable substituted for *n*)

For *d = 1* to *m-1*

{Check whether $L=C_{t,k,1,d}$

If $L=C_{t,k,1,d}$

Find subcarrier $\hat{n}$ satisfying $\lvert h_{k,L,m}\rvert>\lvert h_{k,\hat{n},m}\rvert>\lvert h_{k,j,m}\rvert$ for all $j\in N-\{L\}$

We get the second subcarrier $\hat{n}$ in the average received power list to avoid using the same subcarrier allocation at the same frequency in these channels.

$L=\hat{n}$;}

Then go to (c) to update P_k, N and $C_{k,s,m}$ with *L* instead of *n*.}

B. Scheme 5

This algorithm is an extension of scheme 4. The grey part in scheme 4 has to be changed to:

{Check whether $\left| L - C_{t,k,1,d} \right| < q$

If $\left| L - C_{t,k,1,d} \right| < q$

Find sub-carrier $\hat{n}$ satisfying $\left| h_{k,L,m} \right| > \left| h_{k,n',m} \right| > \left| h_{k,j,m} \right|$ for all $j \in N - \{L\}$

We get the sub-carrier $\hat{n}$ by order in the average received power list to avoid using the same sub-carrier and the adjacent sub-carriers at the same frequency in these channels.

$L = \hat{n}$;}

In this thesis, *q=10*; different value of this parameter can be applied for other cases.

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