Complex Field Network Coding for Multiuser Cooperative Communications

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***Abstract*—Multi-source relay-based cooperative communications can achieve spatial diversity gains, enhance coverage and potentially increase capacity when multiuser detection is used to effect maximum likelihood demodulation. If considered for large networks, traditional relaying entails loss in spectral efficiency that can be mitigated through network coding at the physical layer. These considerations motivate the complex field network coding (CFNC) approach introduced in this paper. Different from network coding over the Galois field, where wireless throughput is limited as the number of sources increases, CFNC always achieves throughput as high as $1/2$ symbol per source per channel use. In addition to improved throughput, CFNC-based relaying achieves full diversity gain regardless of the underlying signal-to-noise-ratio (SNR) and the constellation used. Furthermore, the CFNC approach is general enough to allow for transmissions from sources to a common destination as well as simultaneous information exchanges among sources.**

***Index Terms*—Cooperative communications, multiuser detection, complex field coding, network coding, diversity gain, link-adaptive regeneration.**

I. Introduction

WITHOUT being necessary to pack multiple antennas per terminal, relay-based cooperation can achieve spatial diversity gains, enhance coverage and potentially increase capacity of wireless communication links [12], [13]. Multiuser detection on the other hand, offers jointly optimal demodulation of mutually interfering digital streams arising due to the superposition of multiple packets received from cooperating sources and relay nodes [17]. However, as the network size grows, traditional relay schemes become increasingly bandwidth inefficient. To break through this bandwidth bottleneck, network coding - a technique originally developed for routing in lossless wireline networks - has been recently applied to wireless relay networks [1], [2], [4], [11], [14].

An adaptive network-coded cooperation protocol was proposed in [2] to map the instantaneous network graph to the channel code graph. More recent research efforts focus on exploiting jointly network coding and the broadcast nature of wireless networks, with or without accounting for decoding errors at the relay(s) [4], [9], [11], [16]. Assuming bit-level synchronization, a physical-layer network coding (PNC) was introduced recently in [21] for an additive white Gaussian noise (AWGN) two-way relay channel to map superposition of electromagnetic signals to simple Galois field GF(2^n) additions of digital bit streams.

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Common to all these works is that network coding is applied over the Galois field thus effecting bit-level operations. To further improve network throughput, the present paper develops a complex field network coding (CFNC) approach which entails symbol-level operations at the physical layer. The basic thesis is that CFNC may offer a better fit than Galois field network coding (GFNC) in the wireless regime, primarily because CFNC achieves higher *throughput* than GFNC and PNC schemes while attaining the maximum possible *diversity* gain with multiuser detection.

Specifically, for a cooperative network with N_S sources, N_R relays and one common destination, a setup represented by the triplet $(N_S, N_R, 1)$, we contend that CFNC offers throughput as high as $\frac{1}{2}$ symbol per source per channel use (sym/S/CU). This clearly improves the throughput of GFNC ($\frac{1}{N_S+N_R}$) and that of traditional relying ($\frac{1}{N_S(N_R+1)}$), especially as N_S and N_R grow large. Full diversity can be achieved with CFNC regardless of the SNR and constellation size. In addition to transmissions from N_S sources to a common destination, CFNC enables also efficient *exchange of information* among sources with each achieving full diversity.

The rest of this paper is organized as follows: Section II, describes the model for the $(N_S, 1, 1)$ setup and analyzes diversity performance; generalization to $(N_S, N_R, 1)$ is pursued in Section III and information exchange among sources is considered in Section IV; simulations are described in Section V; and conclusions are summarized in Section VI.

Notation: Upper and lower case bold symbols denote matrices and column vectors, respectively; $(\cdot)^*$ denotes conjugation; $(\cdot)^T$ transpose; $(\cdot)^{\mathcal{H}}$ Hermitian transpose; $\mathcal{CN}(0, \sigma^2)$ the circular symmetric complex Gaussian distribution with zero mean and variance σ^2; $\hat{x}$ the estimate of x; for a random variable γ, $\bar{\gamma} = E\{\gamma\}$ denotes its mean; $\text{Expo}(1/\bar{\gamma})$ the exponential distribution with mean $\bar{\gamma}$; $\text{Gamma}(n, \bar{\gamma})$ the gamma distribution, and specifically with $n = 1$, $\text{Gamma}(1, \bar{\gamma}) = \text{Expo}(1/\bar{\gamma})$; $Q(x) := (1/\sqrt{2\pi}) \int_x^\infty \exp(-t^2/2)dt$ is a monotonically decreasing function of x.

II. CFNC for the $(N_S, 1, 1)$ Cooperative Network

We will first consider the $(2, 1, 1)$ setup in Subsection II-A, generalize it to $(N_S, 1, 1)$ in Subsection II-B and analyze its error performance in Subsection II-C.

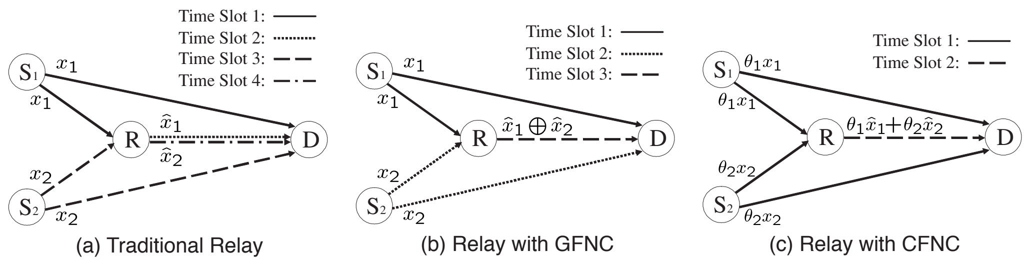

Fig. 1. Relay scheduling schemes.

A. The $(2, 1, 1)$ Paradigm Network

Consider the two-source one-destination wireless relay network depicted in Fig. 1. With a single antenna per node, two sources (S_1 and S_2) send information to the destination (D) directly, and through the relay (R). To avoid interference, sources S_1 and S_2 in traditional relaying transmit over orthogonal channels, e.g., via time division multiple access as in Fig. 1(a). First, S_1 sends symbol x_1 to both R and D during channel use (CU) 1. After detecting x_1 as $\hat{x}_1$, R forwards $\hat{x}_1$ to D in CU 2. Likewise, S_2 sends x_2 directly to D and through R in channel uses (CUs) 3 and 4, respectively. Since a total of 4 channel uses are needed to transmit one symbol per source, the throughput is $1/4$ symbol per source per channel use (sym/S/CU). Note that diversity of order two provided by the Rayleigh fading channels can be collected, in principle, since D receives two copies of x_1 and x_2.

The relay scheme based on GFNC is illustrated in Fig. 1(b). After detecting x_1 and x_2 sent as in traditional relaying during the first two channel uses, R forwards to D in CU 3 the GF coded symbol $\hat{x}_1 \oplus \hat{x}_2$, where $\oplus$ denotes bitwise exclusive XOR operation. Clearly, the throughput of GFNC-based relaying is $1/3$ sym/S/CU. When the channels are error free, D receives one copy of x_1 in CU 1, along with a second copy $x_1 = x_2 \oplus (x_1 \oplus x_2)$ obtained from the GF sum of the received symbols during CUs 2 and 3. Likewise, two copies of x_2 are obtained at D after 3 channel uses, and therefore diversity of order 2 is possible as in traditional relaying.

Next, we introduce the CFNC-based relay scheme which further improves network throughput. As illustrated in Fig. 1(c), R receives *simultaneously* signals $\theta_1 x_1$ and $\theta_2 x_2$ transmitted from S_1 and S_2 in CU 1, where the agreed coefficients θ_1 and θ_2 drawn from the complex field $\mathcal{C}$ will be specified later. The received symbols at R and D after CU 1 are

$$y_{SR} = h_{S_1R}\theta_1 x_1 + h_{S_2R}\theta_2 x_2 + n_{SR} \quad (1)$$
$$y_{SD} = h_{S_1D}\theta_1 x_1 + h_{S_2D}\theta_2 x_2 + n_{SD} \quad (2)$$

where for each pair of subscripts, $h_{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2)$ denotes the channel coefficient and $n_{ij} \sim \mathcal{CN}(0, N_0)$ denotes the AWGN term. The instantaneous and average SNRs are given respectively by $\gamma_{ij} := |h_{ij}|^2\bar{\gamma}$ and $\bar{\gamma}_{ij} := \sigma_{ij}^2\bar{\gamma}$, where $\bar{\gamma} := \mathcal{P}_x/N_0$ and $\mathcal{P}_x$ denotes the average transmit-power of source symbols x which are assumed drawn from a finite alphabet $\mathcal{A}_x$ with cardinality $|\mathcal{A}_x|$. Maximum likelihood (ML) demodulation at the relay yields

$$(\hat{x}_1, \hat{x}_2)_R = \arg\min_{x_1,x_2\in\mathcal{A}_x} ||y_{SR} - h_{S_1R}\theta_1 x_1 - h_{S_2R}\theta_2 x_2||^2. \quad (3)$$

is this a reliable demodulation scheme

Among the different forwarding schemes one can employ at R, we adopt the link-adaptive regenerative (LAR) relaying scheme of [19], where relayed symbols are scaled before being forwarded to the destination. Adapting the scale to the intended source-relay-destination channels, LAR forwarding outperforms existing alternatives in terms of diversity, complexity and power efficiency [19].

With LAR relaying, the input/output (I/O) relationship in CU 2 is given by

$$y_{RD} = h_{RD}\sqrt{\alpha}(\theta_1\hat{x}_1 + \theta_2\hat{x}_2) + n_{RD} \quad (4)$$

where $h_{RD} \sim \mathcal{CN}(0, \sigma_{RD}^2)$, $n_{RD} \sim \mathcal{CN}(0, N_0)$ and α represents a link-adaptive scalar which controls transmit power at R. With $h_{RD}\sqrt{\alpha}$ obtained at D through channel training of the $R \rightarrow D$ link, the ML detector at D using information from two channel uses can be expressed as

$$(\hat{x}_1, \hat{x}_2)_D = \arg\min_{x_1,x_2\in\mathcal{A}_x} \left\{||y_{SD} - h_{S_1D}\theta_1 x_1 - h_{S_2D}\theta_2 x_2||^2 + ||y_{RD} - h_{RD}\sqrt{\alpha}(\theta_1 x_1 + \theta_2 x_2)||^2\right\}.$$

It is important to stress that CFNC in this $(2, 1, 1)$ setup improves throughput to $1/2$ sym/S/CU. Compared to a co-located two-transmit one-receive antennae system, the throughput loss is due to the practical half-duplex constraint of the relay, since R cannot receive and transmit signals using a single antenna over the same channel. In addition to improving throughput, CFNC is also attractive in terms of symbol error probability (SEP), especially when the SNR is sufficiently high as we will see later in Subsection II-C. Three remarks are now in order.

Remark 1: The distinct feature of CFNC is a one-to-one mapping between the ordered pair (x_1, x_2) and $u = \theta_1 x_1 + \theta_2 x_2$, i.e., $\theta_1 x_1 + \theta_2 x_2 \neq \theta_1 x_2 + \theta_2 x_1$, if and only if $x_1 \neq x_2$. This enables detection of both x_1 and x_2 based only on u. Because this property does not hold for the XOR operation $x_1 \oplus x_2$ ($x_1 \oplus x_2 = x_2 \oplus x_1$), GFNC cannot achieve throughput as high as CFNC even when the source bits are added "in the air" with perfect bit-level synchronization as in [21].

Remark 2: Physical layer network coding can also achieve throughput $1/2$ sym/S/CU, but only for the setup considered in [21], where each source is also destination for the information sent by the other source. This two-source relay network shown in Fig. 2(b) can be seen as a sub-network of the two-source CFNC-based relay network with a common destination [cf. Fig. 2(a)], since the broadcast nature of wireless transmissions allows S_1 and S_2 to "hear" R during CU 2. Hence, using CFNC it is possible to achieve high-throughput information

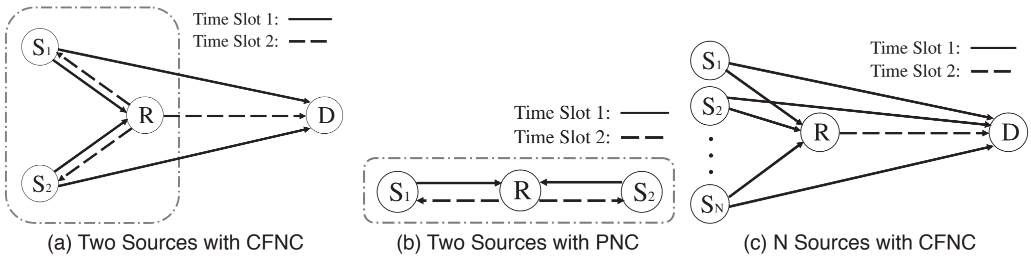

Fig. 2. Comparison between CFNC- and PNC-based networks.

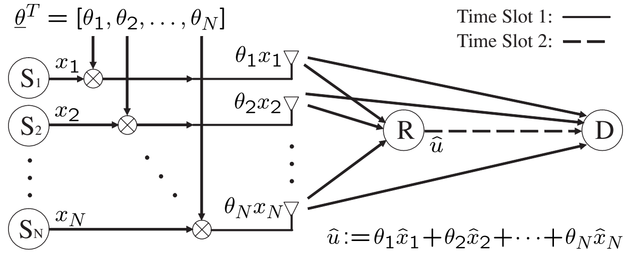

Fig. 3. N-source setup with CFNC.

exchange between S_1 and S_2 as in [21], while sources can also reach a common destination D.

Remark 3: Similar to *all* cooperative relay protocols, physical layer network coding, GFNC and this paper's CFNC-based approach require timing synchronization. This can be acquired using algorithms developed for co-located multi-antenna systems; see e.g., [15], [8, Ch. 11] and references therein. For the special setup in [21], it is possible to bypass this synchronization requirement [10]. Note also that CFNC requires *symbol-level* synchronization which is easier to acquire than *bit-level* synchronization required by GFNC.

B. The $(N_S, 1, 1)$ Cooperative Network

In this section, we generalize the CFNC approach to a setup involving N_S sources ($N_S = N$ here for notational brevity). Upon defining $\mathbf{x} := [x_1, \ldots, x_N]^T$ and $\boldsymbol{\theta}^T := [\theta_1, \ldots, \theta_N]$, the I/O relationships in CU 1 are (see also Fig. 3)

$$y_{SR} = \boldsymbol{\theta}^T \mathbf{H}_{SR} \mathbf{x} + n_{SR} \tag{5}$$

$$y_{SD} = \boldsymbol{\theta}^T \mathbf{H}_{SD} \mathbf{x} + n_{SD} \tag{6}$$

where $\mathbf{H}_{SR} := \text{diag}(h_{S_1R}, \ldots, h_{S_NR})$ and $\mathbf{H}_{SD} := \text{diag}(h_{S_1D}, \ldots, h_{S_ND})$. In CU 2, the relay forwards information from the sources to D with scaled power to obtain

$$y_{RD} = h_{RD}\sqrt{\alpha}\boldsymbol{\theta}^T \hat{\mathbf{x}}_R + n_{RD} \tag{7}$$

where the $N \times 1$ symbol vector $\hat{\mathbf{x}}_R$ is obtained at R using ML detection as

$$\hat{\mathbf{x}}_R = \arg\min_{\mathbf{x}\in\mathcal{A}_\mathbf{x}} ||y_{SR} - \boldsymbol{\theta}^T \mathbf{H}_{SR}\mathbf{x}||^2. \tag{8}$$

The ML detector at D based on data received in channel uses 1 and 2 yields

$$\hat{\mathbf{x}}_D = \arg\min_{\mathbf{x}\in\mathcal{A}_\mathbf{x}} \Big\{ ||y_{SD} - \boldsymbol{\theta}^T \mathbf{H}_{SD}\mathbf{x}||^2 + ||y_{RD} - h_{RD}\sqrt{\alpha}\boldsymbol{\theta}^T\mathbf{x}||^2 \Big\}.$$

Hence, over only two channel uses, information from N sources arrives at D along with an extra copy received from R.

Critical to CFNC is the design of $\boldsymbol{\theta}^T$ in (5) and (6). Before transmission in CU 1, the source signal x_i from S_i is multiplied by θ_i, the i^{th} entry of $\boldsymbol{\theta}^T := [\theta_1, \theta_2, \ldots, \theta_N]$, $i = 1, \ldots, N$. Vector $\boldsymbol{\theta}$ is assumed available at every node in the network. Among the different choices for $\boldsymbol{\theta}$, we adopt the one based on linear constellation precoding designed for multiple-input multiple-output (MIMO) systems in [8]; see also [5], [6], and [20]. For $N = 2^k$, the entries of $\boldsymbol{\theta}$ in this design are given by $\theta_i = e^{j\pi(4n-1)(i-1)/(2N)}$; and for $N = 3 \times 2^k$, by $\theta_i = e^{j\pi(6n-1)(i-1)/(3N)}$ for any $n = 1, \ldots, N$. The major difference between precoded MIMO transmissions involved with co-located multi-antenna systems and those used here for CFNC is due to possible decoding errors at the relay which complicate diversity analysis of the CFNC-based network, the subject undertaken next.

how to get the diversity gain

C. Performance Analysis

The diversity gain G_d is defined as the negative exponent in the average SEP when the average SNR tends to infinity, that is $E[P] \overset{\bar{\gamma}\to\infty}{\approx} (G_c\bar{\gamma})^{-G_d}$, where G_c denotes the coding gain. For symbol-by-symbol demodulation of uncoded transmissions, G_c depends on the constellation and the transmit-power. On the other hand, G_d depends on the degrees of freedom provided by the underlying fading channels.

With $\sigma_{SR}^2 := E[|h_{S_iR}|^2]$, $\sigma_{SD}^2 := E[|h_{S_iD}|^2]$, and $\sigma_{RD}^2 := E[|h_{RD}|^2]$ denoting the per-link variances, the objective of this section is to establish that:

Proposition 1: *The $(N, 1, 1)$ wireless cooperative network with CFNC can achieve throughput $1/2$ sym/S/CU and full diversity with ML multiuser detection for any N and any SNR triplet $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2)$.*

The throughput claim in Proposition 1 has been argued in Section II-B. The proof of the diversity claim will be carried through by successively bounding the SEP. Because symbols $u = \boldsymbol{\theta}^T\mathbf{x}$ are drawn from a constellation $\mathcal{A}_u$, with cardinality $|\mathcal{A}_u| = |\mathcal{A}_x|^N$, the SEP for u (P^u) should upper bound that for x_i (P^{x_i}); i.e., $P^u \geq P^{x_i}\ \forall\ i = 1, \ldots, N$. Hence, to prove that $E[P^{x_i}]$ achieves full diversity, which is 2 for the $(2, 1, 1)$ setup, it suffices to prove that $E[P^u]$ achieves full diversity.

Let power $\mathcal{P}_x = 1$, $d^{\min}$ ($d^{\max}$) denote the minimum (maximum) Euclidean distance in the constellation of $\boldsymbol{\theta}^T\mathbf{x}$, i.e., in $\mathcal{A}_u$, and $d_{SR}^{\min}$ ($d_{SD}^{\min}$) the minimum Euclidean distance in the constellation of $\boldsymbol{\theta}^T\mathbf{H}_{SR}\mathbf{x}$ (respectively $\boldsymbol{\theta}^T\mathbf{H}_{SD}\mathbf{x}$). We will find it useful to consider a "virtual" $S \rightarrow R$ link with I/O relationship

$$\tilde{y}_{SR} = \tilde{h}_{SR}\boldsymbol{\theta}^T\mathbf{x} + \tilde{n}_{SR} = \tilde{h}_{SR}u + \tilde{n}_{SR} \quad (9)$$

where $\tilde{h}_{SR}$ has an arbitrary phase known at R and amplitude $|\tilde{h}_{SR}| := d_{SR}^{\min}/d^{\max}$. Note that the *maximum* Euclidean distance of the received constellation in (9) is $|\tilde{h}_{SR}|d^{\max}\sqrt{\mathcal{P}_x} = d_{SR}^{\min}$, which equals the *minimum* Euclidean distance of the received constellation in (5). Thus, the SEP of the "virtual" scalar channel ($\tilde{P}_{SR}^u$) upper bounds that of the original $S \rightarrow R$ link (P_{SR}^u). Making use of the union bound, $\tilde{P}_{SR}^u$ can be bounded as

$$\tilde{P}_{SR}^u \leq (M-1)Q\left(\sqrt{2\tilde{\gamma}_{SR}}\right) \leq (M-1)\exp(-\tilde{\gamma}_{SR}) \quad (10)$$

where $\tilde{\gamma}_{SR} := |\tilde{h}_{SR}|^2(d^{\min}/2)^2\bar{\gamma}$, $M := |\mathcal{A}_u|$ and the last inequality follows from the Chernoff bound.

Similarly, we can also consider a "virtual" $S \rightarrow D$ channel with I/O relationship

$$\tilde{y}_{SD} = \tilde{h}_{SD}\boldsymbol{\theta}^T\mathbf{x} + \tilde{n}_{SD} = \tilde{h}_{SD}u + \tilde{n}_{SD} \quad (11)$$

where $\tilde{h}_{SD}$ has an arbitrary phase known at D, amplitude $|\tilde{h}_{SD}| := d_{SD}^{\min}/d^{\max}$ and $\tilde{\gamma}_{SD} := |\tilde{h}_{SD}|^2(d^{\min}/2)^2\bar{\gamma}$. Consider now detecting $\mathbf{x}$ at D based on y_{RD} received from the relay [cf. (7)] and the virtual $\tilde{y}_{SD}$ given by (11). It can be readily verified that ML detection using y_{RD} and $\tilde{y}_{SD}$ is equivalent to maximum ratio combining (MRC) and yields

$$\begin{aligned}\hat{\mathbf{x}}_D = \arg\min_{\mathbf{x}\in\mathcal{A}_\mathbf{x}} &|\tilde{h}_{SD}^*\tilde{y}_{SD} + \sqrt{\alpha}h_{RD}^*y_{RD}\\ &-(|\tilde{h}_{SD}|^2 + \alpha|h_{RD}|^2)\boldsymbol{\theta}^T\mathbf{x}|^2. \quad (12)\end{aligned}$$

Since $\tilde{P}_{SD}^u \geq P_{SD}^u$, the SEP $\tilde{P}^u$ of the MRC detection in (12) upper bounds the SEP P^u of the original system. We wish to prove that $\tilde{P}^u$ exhibits full diversity. Toward this goal, let P_c^u (P_e^u) denote the SEP at D conditioned on the fact that R forwards a correct (correspondingly erroneous) u. Considering that R may correctly decode $\mathbf{x}$ or not (with probability $1 - \tilde{P}_{SR}^u$ and $\tilde{P}_{SR}^u$ respectively), we obtain $\tilde{P}^u = (1-\tilde{P}_{SR}^u)P_c^u + \tilde{P}_{SR}^u P_e^u \leq P_c^u + \tilde{P}_{SR}^u P_e^u$.

Our next step is to bound the instantaneous virtual SNRs ($\tilde{\gamma}_{SR}$, $\tilde{\gamma}_{SD}$) using the minimum SNRs of the actual links, namely $\gamma_{SR}^{\min} := \min(\gamma_{S_1R},\ldots,\gamma_{S_NR})$ and $\gamma_{SD}^{\min} := \min(\gamma_{S_1D},\ldots,\gamma_{S_ND})$, as asserted in the following lemma (see Appendix A for the proof).

Lemma 1: *It holds that* $\Delta_{SR}\gamma_{SR}^{\min} \leq \tilde{\gamma}_{SR} \leq \gamma_{SR}^{\min}$*, where* Δ_{SR} *is a random variable independent of* $\gamma_{SR}^{\min}$ *satisfying* $\Pr(\Delta_{SR} > 0) = 1$*; in addition, it holds that* $\Delta_{SD}\gamma_{SD}^{\min} \leq \tilde{\gamma}_{SD} \leq \gamma_{SD}^{\min}$*, where* Δ_{SD} *is a random variable independent of* $\gamma_{SD}^{\min}$ *satisfying* $\Pr(\Delta_{SD} > 0) = 1$.

Lemma 1 allows us to further upper bound $\tilde{P}_{SR}^u$ in (10) as $\tilde{P}_{SR}^u \leq (M-1)\exp(-\Delta_{SR}\gamma_{SR}^{\min})$; and hence

$$\tilde{P}^u \leq P_c^u + (M-1)\exp(-\Delta_{SR}\gamma_{SR}^{\min})P_e^u. \quad (13)$$

Also from Lemma 1 and the union bound, we obtain

$$P_c^u \leq (M-1)Q\left[\sqrt{2(\Delta_{SD}\gamma_{SD}^{\min} + \alpha\gamma_{RD})}\right] \quad (14)$$

where $\gamma_{RD} := |h_{RD}|^2(d^{\min}/2)^2\bar{\gamma}$. To bound P_e^u in (13), note that if there is a detection error at R, then P_e^u can be upper bounded by the worst case which corresponds to having the decoded symbol at R as the farthest constellation point (i.e., at distance $d^{\max}$) from the actual symbol sent from the sources. (A detailed proof of this claim in a different context can be found in [18].) Based on this worst case, using Lemma 1 and the union bound again, we arrive at

$$P_e^u \leq (M-1)Q\left\{\frac{\sqrt{2}\left[\Delta_{SD}\gamma_{SD}^{\min} - \alpha\gamma_{RD}2\beta\right]}{\sqrt{\Delta_{SD}\gamma_{SD}^{\min} + \alpha\gamma_{RD}2\beta}}\right\} \quad (15)$$

where $\beta := d^{\max}/d^{\min}$. The α chosen to control transmit-power at R, affects critically the SEP [19]. A simple diversity-achieving α is the following one introduced in [19]

$$\alpha := \frac{\min\{\tilde{\gamma}_{SR}, \bar{\gamma}_{RD}\}}{\bar{\gamma}_{RD}} \quad (16)$$

where $\bar{\gamma}_{RD} := \sigma_{RD}^2(d^{\min}/2)^2\bar{\gamma}$. Notice that this α depends only on the instantaneous channel state information (CSI) of the $S_i \rightarrow R$ links and the average CSI $\bar{\gamma}_{RD}$. Intuitively, α in (16) is chosen small when the $S_i \rightarrow R$ link is unreliable (i.e., $\tilde{\gamma}_{SR}$ is very small); and large when the $S_i \rightarrow R$ link is reliable.

Combining bounds (13)-(15), $E[P^u]$ can be upper bounded as $E[P^u] \leq E[P_1^u] + E[P_2^u]$, where

$$\begin{aligned}P_1^u &= (M-1)Q\left[\sqrt{2(\Delta_{SD}\gamma_{SD}^{\min} + \alpha\gamma_{RD})}\right] \quad (17)\\ P_2^u &= (M-1)^2\exp(-\Delta_{SR}\gamma_{SR}^{\min})\\ &\quad\times Q\left\{\frac{\sqrt{2}\left[\Delta_{SD}\gamma_{SD}^{\min} - \alpha\gamma_{RD}2\beta\right]}{\sqrt{\Delta_{SD}\gamma_{SD}^{\min} + \alpha\gamma_{RD}2\beta}}\right\}. \quad (18)\end{aligned}$$

To establish that $E[P^u]$ achieves full diversity, we rely on the following lemma to show that both $E[P_1^u]$ and $E[P_2^u]$ achieve full diversity (see Appendix B for the proof).

Lemma 2: *Consider the symbol error probability function* P_e *satisfying*

$$P_e \leq \exp(-\eta_e'\gamma_e)Q\left[\frac{\sqrt{2}(\eta_c\gamma_c - \eta_e\gamma_e)}{\sqrt{\eta_c\gamma_c + \eta_e\gamma_e}}\right]$$

where $\gamma_c \sim \text{Gamma}(n_c, \bar{\gamma})$ *and* $\gamma_e \sim \text{Gamma}(n_e, \bar{\gamma})$ *are independent; and* η_c, η_e *and* η_e' *are nonnegative random variables independent of* γ_c *and* γ_e*. If the probability density functions* $p(\eta_c)$, $p(\eta_e)$ *and* $p(\eta_e')$ *do not depend on* $\bar{\gamma}$*, and* $\Pr(\eta_c > 0) = \Pr(\eta_e > 0) = \Pr(\eta_e' > 0) = 1$*, then the average* P_e *is bounded as* $E[P_e] \leq \kappa\bar{\gamma}^{-(n_c+n_e)}$*, where* $\kappa := E[k(\eta_c, \eta_e, \eta_e')] > 0$ *is a constant not dependent on* $\bar{\gamma}$*; hence,* $E[P_e]$ *achieves diversity of order* $n_c + n_e$.[1]

We next derive pertinent bounds for (17) and (18) to match the requirements in Lemma 2. Upon defining the i.i.d. random variables $\gamma_{ij}^{\text{norm}} \sim \text{Expo}(1/\bar{\gamma}) = \text{Gamma}(1, \bar{\gamma})$ for any pair

[1]We define $\gamma_c \sim \text{Gamma}(0, \bar{\gamma})$ if $\gamma_c = 0$ and $\gamma_e \sim \text{Gamma}(0, \bar{\gamma})$ if $\gamma_e = 0$; so Lemma 2 holds for any $n_c \geq 0$ and $n_e \geq 0$.

of subscripts (i,j), the SNR terms of P_1^u in (17) can be expressed as [cf. (16)]

$$\Delta_{SD}\gamma_{SD}^{\min}+\alpha\gamma_{RD} = \Delta_{SD}\frac{\sigma_{SD}^2}{N}\gamma_{SD}^{\text{norm}}+\min\left(\frac{\tilde{\gamma}_{SR}}{\bar{\gamma}_{RD}},1\right)\left(\frac{d^{\min}}{2}\right)^2\sigma_{RD}^2\gamma_{RD}^{\text{norm}}.$$

As a result, P_1^u can be upper bounded as $P_1^u \leq (M-1)Q\left[\sqrt{2\eta_c\gamma_c}\right]$ where $\gamma_c = \gamma_{SD}^{\text{norm}}+\gamma_{RD}^{\text{norm}}$ and $\eta_c = \min\left\{\Delta_{SD}\frac{\sigma_{SD}^2}{N},\ \min\left(\frac{\Delta_{SR}\gamma_{SR}^{\min}}{\bar{\gamma}_{RD}},1\right)\left(\frac{d^{\min}}{2}\right)^2\sigma_{RD}^2\right\}$. Thus, $\gamma_c \sim \text{Gamma}(2,\bar{\gamma})$ and $\Pr(\eta_c>0)=1$ from Lemma 1. Since the coefficient $(M-1)$ can be absorbed by the constant κ, Lemma 2 applies to P_1^u with $\gamma_e=0$, $n_c=2$ and $n_e=0$; and therefore, $E[P_1^u]$ achieves diversity 2 ($=n_c+n_e$).

For P_2^u, since the power scale in (16) satisfies $\alpha \leq \frac{\tilde{\gamma}_{SR}}{\bar{\gamma}_{RD}} \leq \frac{\gamma_{SR}^{\min}}{\bar{\gamma}_{RD}}$, we obtain

$$P_2^u \leq (M-1)^2\exp(-\eta_e'\gamma_e)Q\left[\frac{\sqrt{2}(\eta_c\gamma_c-\eta_e\gamma_e)}{\sqrt{\eta_c\gamma_c+\eta_e\gamma_e}}\right] \quad (19)$$

and Lemma 2 applies with $\gamma_c=\gamma_{SD}^{\text{norm}}$, $\gamma_e=\gamma_{SR}^{\text{norm}}$, $\eta_c=\Delta_{SD}\frac{\sigma_{SD}^2}{N}$, $\eta_e=2\beta\frac{\gamma_{RD}}{\bar{\gamma}_{RD}}\frac{\sigma_{SR}^2}{N}$, $\eta_e'=\Delta_{SR}\frac{\sigma_{SR}^2}{N}$, $n_c=1$ and $n_e=1$. Arguing as before, it follows that $E[P_2^u]$ also achieves diversity 2. Thus, full diversity 2 is exhibited by $E[P^u]$ and hence by the average SEP corresponding to each source and any triplet $(\sigma_{SR}^2,\sigma_{SD}^2,\sigma_{RD}^2)$. This completes the proof of Proposition 1.

At this point it is appropriate to clarify the merits of CFNC relative to GFNC with regards to throughput and diversity.

Remark 4: Instrumental to CFNC's throughput improvement is the $N\times 1$ linear constellation precoding vector $\boldsymbol{\theta}$. Its judicious design ensures that from the linear combination $u=\boldsymbol{\theta}^T\mathbf{x}$ it is possible to *uniquely* recover $\mathbf{x}$. This holds true for any N and $\mathbf{x}$ drawn from a finite alphabet (lattice); see [8, pp. 70-79], [6] and references therein.[2] Notwithstanding, for GFNC to be able to match CFNC's throughput (enabled by the linear combination of symbols from different sources received simultaneously at R and D), it should be feasible to uniquely recover each $x_i, i=1,\ldots,N$, from the XOR aggregate $x_1\oplus x_2\oplus\cdots\oplus x_N$, which is generally impossible. The only exception is through physical network coding in the special setup considered in [21], where $N=2$ and both sources act also as destinations but a separate destination is absent. In the presence of a separate destination, CFNC will still have higher throughput even with $N=2$ sources.

Remark 5: With regards to diversity, one may be tempted to think that even source-relay channels with independent, identically distributed phases could enable the diversity as in multi-input single-output random fading channels carrying a common symbol. Apart from the fact that channels maybe correlated, this is impossible in the present high-throughput setup because the sources transmit different symbols x_i. The diversity in CFNC comes from the extra copies provided at the destination by the relays. In fact, key enabler of CFNC's maximum diversity is the link adaptive power scaling of the linearly precoded symbol u that is decoded at the relay and forwarded to the destination. The role of α in enabling the diversity of general decode-and-forward relay transmissions has been documented in [19]. Here it is applied to the linearly precoded symbol based on which the destination can uniquely recover the extra copies of each x_i and combine them with their counterparts received from the source-destination channels to collect (via multiuser detection) the maximum possible diversity. Note again that unique recovery of individual symbols from their XOR superposition is generally impossible with GFNC.

[2]With binary modulations and $N=2$, linear constellation precoding can be also interpreted as constellation rotation; but this interpretation does not apply in general.

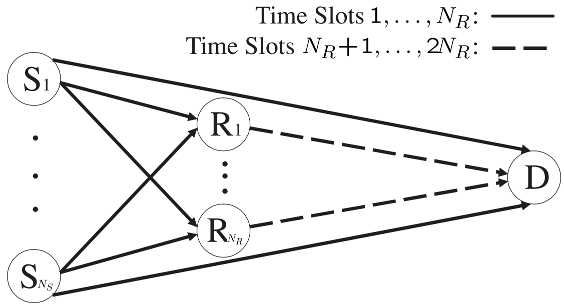

Fig. 4. An $(N_S,N_R,1)$ CFNC-based network with PR.

III. The $(N_S,N_R,1)$ Cooperative Network

The objective of this section is to establish that a general CFNC-based $(N_S,N_R,1)$ network can attain symbol rate $1/2$ sym/S/CU and full diversity N_R+1. Two alternatives will be considered for the multi-relay operation, starting with that of parallel relays (PR).

A. Parallel Relays

With reference to Fig. 4, consider N_S sources transmitting with CFNC to N_R relays and the destination D over N_R successive channel uses during which h_{SR}'s and h_{SD}'s are assumed to remain invariant. Defining $\mathbf{H}_{SR_j}:=\text{diag}(h_{S_1R_j},\cdots,h_{S_{N_s}R_j})$, $\mathbf{H}_{SD}:=\text{diag}(h_{S_1D},\cdots,h_{S_{N_s}D})$ and $\mathbf{x}(t):=[x_1(t),\ldots,x_{N_S}(t)]^T$, the I/O relationships in these first N_R channel uses are

$$y_{SR_j}(t) = \boldsymbol{\theta}_S^T\mathbf{H}_{SR_j}\mathbf{x}(t)+n_{SR_j}(t) \quad (20)$$
$$y_{SD}(t) = \boldsymbol{\theta}_S^T\mathbf{H}_{SD}\mathbf{x}(t)+n_{SD}(t) \quad (21)$$

where $t=1,\ldots,N_R$, $j=1,\ldots,N_R$, and $\boldsymbol{\theta}_S$ is an $N_S\times 1$ vector designed as in Subsection II-B.

After N_R channel uses, relay R_j detects $\hat{\mathbf{x}}_j(t) = \underset{\mathbf{x}(t)}{\text{argmin}}||y_{SR_j}(t)-\boldsymbol{\theta}_S^T\mathbf{H}_{SR_j}\mathbf{x}(t)||^2$ and forwards this demodulated symbol with scaling coefficient α_j in CU N_R+j. The I/O relationship is

$$y_{R_jD}=\sqrt{\alpha_j}h_{R_jD}\boldsymbol{\theta}_R^T\hat{\mathbf{x}}_j+n_{R_jD}', \quad j=1,\ldots,N_R \quad (22)$$

where $\hat{\mathbf{x}}_j:=[\hat{\mathbf{x}}_j^T(1),\ldots,\hat{\mathbf{x}}_j^T(N_R)]^T$, and $\boldsymbol{\theta}_R$ is an $N_SN_R\times 1$ vector designed as in Subsection II-B.

Since N_R symbols are transmitted per source over $2N_R$ channel uses, the symbol rate is clearly $1/2$ sym/S/CU. The

ML detector at D after $2N_R$ channel uses yields

$$\hat{\mathbf{x}}_D = \underset{\mathbf{x}'}{\operatorname{argmin}} \left\{ \sum_{t=1}^{N_R} ||y_{SD}(t) - \boldsymbol{\theta}_S^T \mathbf{H}_{SD}\mathbf{x}(t)||^2 + \sum_{j=1}^{N_R} ||y_{R_jD} - \sqrt{\alpha_j} h_{R_jD} \boldsymbol{\theta}_R^T \mathbf{x}'||^2 \right\}$$

where $\mathbf{x}' := [\mathbf{x}^T(1), \ldots, \mathbf{x}^T(N_R)]^T$.

B. Performance Analysis for Parallel Relays

Let $d_S^{\min}$ ($d_R^{\min}$) and $d_S^{\max}$ ($d_R^{\max}$) denote the minimum and maximum Euclidean distance in the constellation of $\boldsymbol{\theta}_S^T\mathbf{x}(t)$ (respectively $\boldsymbol{\theta}_R^T\mathbf{x}'$), and $d_{SR_j}^{\min}$ ($d_{SD}^{\min}$) the minimum Euclidean distance in the constellation of $\boldsymbol{\theta}_S^T\mathbf{H}_{SR}\mathbf{x}(t)$ (respectively $\boldsymbol{\theta}_S^T\mathbf{H}_{SD}\mathbf{x}(t)$), when $\mathcal{P}_x = 1$. For each R_j, consider again a "virtual" $S \to R_j$ link with I/O relationship

$$\tilde{y}_{SR_j}(t) = \tilde{h}_{SR_j}\boldsymbol{\theta}_S^T\mathbf{x}(t) + \tilde{n}_{SR_j}(t) \tag{23}$$

where $\tilde{h}_{SR_j}$ has an arbitrary phase known at R_j and amplitude $|\tilde{h}_{SR_j}| := d_{SR_j}^{\min}/d_S^{\max}$, $j = 1, \ldots, N_R$. The SEP of each "virtual" $S \to R_j$ link, i.e., $\tilde{P}_{SR_j}^u$, is upper bounded as $\tilde{P}_{SR_j}^u \le (M-1)Q\left(\sqrt{2\tilde{\gamma}_{SR_j}}\right)$, where $\tilde{\gamma}_{SR_j} := |\tilde{h}_{SR_j}|^2 (d_S^{\min}/2)^2\bar{\gamma}$. At the end of CU N_R, the SEP for $v := \boldsymbol{\theta}_R^T\mathbf{x}'$ at R_j can be upper bounded as $\tilde{P}_{SR_j}^v \le N_R\tilde{P}_{SR_j}^u \le N_R(M-1)\exp(-\Delta_{SR_j}\gamma_{SR_j}^{\min})$, where Δ_{SR_j} and $\gamma_{SR_j}^{\min}$ play the roles of Δ_{SR} and $\gamma_{SR}^{\min}$ in the $(N,1,1)$ setup.

To lower bound the SEP at D, consider further a "virtual" $S \to D$ link with I/O relationship

$$\tilde{y}_{SD} = \tilde{h}_{SD}\boldsymbol{\theta}_R^T\mathbf{x}' + \tilde{n}_{SD} \tag{24}$$

where $\tilde{h}_{SD}$ has an arbitrary phase known at D and amplitude $|\tilde{h}_{SD}| := d_{SD}^{\min}/d_R^{\max}$. Then we can upper bound P^v at D by $P^v \le \tilde{P}^v$, where $\tilde{P}^v$ denotes the SEP of the following MRC [cf. (22) and (24)]

$$\hat{\mathbf{x}}_D = \underset{\mathbf{x}'}{\operatorname{argmin}} ||\tilde{h}_{SD}^*\tilde{y}_{SD} + \sum_{j=1}^{N_R}\sqrt{\alpha_j}h_{R_jD}^* y_{R_jD} - (|\tilde{h}_{SD}|^2 + \sum_{j=1}^{N_R}\alpha_j|h_{R_jD}|^2)\boldsymbol{\theta}_R^T\mathbf{x}'||^2.$$

Following steps analogous to those in [18] and invoking the union bound for the worst case constellation as in the $(N,1,1)$ setup, $\tilde{P}^v$ is upper bounded as

$$\tilde{P}^v = \sum_{\varepsilon=0}^{N_R}\sum_{j=1}^{\binom{N_R}{\varepsilon}} \left\{ \prod_{k=1}^{\varepsilon} P_{SR_{E_k^j}}^v \prod_{l=1}^{N_R-\varepsilon}(1 - P_{SR_{C_l^j}}^v) P^v(j,\varepsilon) \right\} \le \sum_{\varepsilon=0}^{N_R}\sum_{j=1}^{\binom{N_R}{\varepsilon}} \left\{ (N_RM-1)^{\varepsilon+1}\exp\left(-\sum_{k=1}^{\varepsilon}\Delta_{SR_{E_k^j}}\gamma_{SR_{E_k^j}}^{\min}\right) Q\left[\frac{\sqrt{2}(\Delta_{SD}\gamma_{SD}^{\min} + \sum_{l=1}^{N_R-\varepsilon}\alpha_{C_l^j}\gamma_{R_{C_l^j}D} - 2\beta\sum_{k=1}^{\varepsilon}\alpha_{E_k^j}\gamma_{R_{E_k^j}D})}{\sqrt{\Delta_{SD}\gamma_{SD}^{\min} + \sum_{l=1}^{N_R-\varepsilon}\alpha_{C_l^j}\gamma_{R_{C_l^j}D} + 2\beta\sum_{k=1}^{\varepsilon}\alpha_{E_k^j}\gamma_{R_{E_k^j}D}}}\right]\right\} \tag{25}$$

where ε is used to index the relays forwarding erroneously detected symbols to D; E^j (C^j) is a set of ε ($N_R - \varepsilon$) distinct elements from the set $\{1, 2, \ldots, N_R\}$; E_k^j (C_l^j) is the k^{th} (l^{th}) element of E^j (C^j), $E^j \cup C^j = \{1, 2, \ldots, N_R\}$, $E^j \cap C^j = \phi$ and for any $j \neq j'$, $E^j \neq E^{j'}$, $C^j \neq C^{j'}$.

Here we choose $\alpha_j := \frac{\min\{\tilde{\gamma}_{SR_j}, \bar{\gamma}_{R_jD}\}}{\bar{\gamma}_{R_jD}}$, where $\bar{\gamma}_{R_jD} := \sigma_{R_jD}^2(d_R^{\min}/2)^2\bar{\gamma}$, $j = 1, \ldots, N_R$. For each pair (ε, j), following arguments similar to those in the previous section, terms inside the curly brackets of (25) can be bounded further. We first lower bound

$$\Delta_{SD}\gamma_{SD}^{\min} + \sum_{l=1}^{N_R-\varepsilon}\alpha_{C_l^j}\gamma_{R_{C_l^j}D} \ge \eta_c\gamma_c \tag{26}$$

where $\gamma_c = \gamma_{SD}^{\text{norm}} + \sum_{l=1}^{N_R-\varepsilon}\gamma_{R_{C_l^j}D}^{\text{norm}}$, $\gamma_c \sim \text{Gamma}(N_R - \varepsilon + 1, \bar{\gamma})$, and

$$\eta_c = \min\left\{\Delta_{SD}\frac{\sigma_{SD}^2}{N_S}, \min_{l=1,\ldots,N_R-\varepsilon}\left[\min\left(\Delta_{SR_{C_l^j}}\frac{\gamma_{SR_{C_l^j}}^{\min}}{\bar{\gamma}_{R_{C_l^j}D}}, 1\right)\left(\frac{d_R^{\min}}{2}\right)^2\sigma_{R_{C_l^j}D}^2\right]\right\}.$$

Then, we upper bound $2\beta\sum_{k=1}^{\varepsilon}\alpha_{E_k^j}\gamma_{R_{E_k^j}D} \le \eta_e\gamma_e$, where $\gamma_e = \sum_{k=1}^{\varepsilon}\gamma_{SR_{E_k^j}}^{\text{norm}}$, $\gamma_e \sim \text{Gamma}(\varepsilon, \bar{\gamma})$ and $\eta_e = 2\beta\max_{k=1,\ldots,\varepsilon}\left\{\sigma_{SR_{E_k^j}}^2\gamma_{R_{E_k^j}D}/(\bar{\gamma}_{R_{E_k^j}D}N_S)\right\}$. At last, we lower bound $\sum_{k=1}^{\varepsilon}\Delta_{SR_{E_k^j}}\gamma_{SR_{E_k^j}}^{\min} \ge \eta_e'\gamma_e$, where $\eta_e' = \min_{k=1,\ldots,\varepsilon}\left\{\Delta_{SR_{E_k^j}}\sigma_{SR_{E_k^j}}^2/N_S\right\}$. This allows application of Lemma 2 with $n_c = N_R - \varepsilon + 1$ and $n_e = \varepsilon$, to obtain

$$P^v \le \sum_{\varepsilon=0}^{N_R}\sum_{j=1}^{\binom{N_R}{\varepsilon}}(N_RM-1)^{\varepsilon+1} \times \exp(-\eta_e'\gamma_e)Q\left[\frac{\sqrt{2}(\eta_c\gamma_c - \eta_e\gamma_e)}{\sqrt{\eta_c\gamma_c + \eta_e\gamma_e}}\right]. \tag{27}$$

Lemma 1 guarantees that the parameters in (27) satisfy the conditions in Lemma 2, from which we deduce that $E[P^v]$ achieves full diversity $N_R + 1$, since the summations and coefficients in (27) can be absorbed in the constant κ of Lemma 2.

Summarizing the results pertaining to PR, we have established that:

Proposition 2: *The $(N_S, N_R, 1)$ wireless cooperative network with CFNC and PR achieves throughput $1/2$ sym/S/CU and full diversity with ML demodulation for each source and any triplet (σ_{SR}^2,σ_{SD}^2,σ_{RD}^2).*

C. Relay Selection

Instead of parallel relays, this subsection will pursue CFNC for an $(N_S, N_R, 1)$ network with a single relay selected per block. Simulations will confirm that relay selection (RS) outperforms parallel relays (PR) over the low-moderate SNR range, where diversity benefits may not be pronounced.

As its name suggests, D in RS selects one out of N_R relays to serve during each block.[3] As mentioned before, D has available the products $h_{R_jD}\sqrt{\alpha_j}$ for $j = 1,\dots,N_R$. From these products, D selects a "winner relay" per block as $R_j = \arg\max_{j=1,\dots,N_R} |h_{R_jD}\sqrt{\alpha_j}| = \arg\max_{j=1,\dots,N_R} \alpha_j\gamma_{R_jD}$; and feeds back its index to all relays. Subsequently, transmission follows exactly the steps in the $(N_S,1,1)$ setup of Section II with R_w serving as the only relay.

Based on $\alpha_w\gamma_{R_wD} := \max_{j=1,\dots,N_R} \alpha_j\gamma_{R_jD}$, one can analyze the SEP along the lines used in Section II to arrive at $E[P^u] \le E[P_1^u] + E[P_2^u]$, where [cf. (17) and (18)]

$$P_1^u = (M-1)Q\left[\sqrt{2(\Delta_{SD}\gamma_{SD}^{\min} + \alpha_w\gamma_{R_wD})}\right] \quad (28)$$

$$P_2^u = (M-1)^2\exp(-\tilde{\gamma}_{SR_w}) \, Q\left[\frac{\sqrt{2}\left(\Delta_{SD}\gamma_{SD}^{\min} - 2\beta\alpha_w\gamma_{R_wD}\right)}{\sqrt{\Delta_{SD}\gamma_{SD}^{\min} + 2\beta\alpha_w\gamma_{R_wD}}}\right]. \quad (29)$$

Since $\alpha_w\gamma_{R_wD} \ge \frac{1}{N_R}\sum_{j=1}^{N_R}\alpha_j\gamma_{R_jD}$, P_1^u is upper bounded as $P_1^u \le (M-1)Q\left[\sqrt{2\eta_c\gamma_c}\right]$, where $\gamma_c = \gamma_{SD}^{\text{norm}} + \sum_{j=1}^{N_R}\gamma_{R_jD}^{\text{norm}}$, $\gamma_c \sim \text{Gamma}(N_R+1,\bar{\gamma})$ and

$$\eta_c = \min\left\{\Delta_{SD}\frac{\sigma_{SD}^2}{N_S}, \min_{j=1,\dots,N_R}\left[\min\left(\frac{\Delta_{SR_j}\gamma_{SR_j}^{\min}}{\bar{\gamma}_{R_jD}},1\right)\left(\frac{d^{\min}}{2}\right)^2\frac{\sigma_{R_jD}^2}{N_R}\right]\right\}. \quad (30)$$

Therefore, Lemma 2 applies to P_1^u in (28) with $\gamma_e = 0$, $n_c = N_R+1$, $n_e = 0$ and establishes that $E[P_1^u]$ achieves diversity N_R+1.

For P_2^u, notice first that if $\tilde{\gamma}_{SR_w} \ge \bar{\gamma}_{R_wD}$, it holds that $P_2^u \le (M-1)^2\exp(-\bar{\gamma}_{R_wD})$ and full diversity is ensured when $\bar{\gamma}$ (or say $\bar{\gamma}_{R_wD}$) is large enough. Otherwise, if $\tilde{\gamma}_{SR_w} < \bar{\gamma}_{R_wD}$, we have $\alpha_w = \tilde{\gamma}_{SR_w}/\bar{\gamma}_{R_wD}$ and

$$\alpha_w\gamma_{R_wD} \ge \left(\sum_{j\in J_1}\frac{\gamma_{R_jD}}{N_R} + \sum_{j\in J_2}\frac{\tilde{\gamma}_{SR_j}}{\bar{\gamma}_{R_jD}}\frac{\gamma_{R_jD}}{N_R}\right)$$

$$\alpha_w\gamma_{R_wD} \le \left(\sum_{j\in J_1}\gamma_{R_jD} + \sum_{j\in J_2}\frac{\tilde{\gamma}_{SR_j}}{\bar{\gamma}_{R_jD}}\gamma_{R_jD}\right)$$

where $J_1 := \{j : j \in [1,\dots,N_R], \tilde{\gamma}_{SR_j} \ge \bar{\gamma}_{R_jD}\}$ and $J_2 := \{j : j \in [1,\dots,N_R], \tilde{\gamma}_{SR_j} < \bar{\gamma}_{R_jD}\}$. As a result, Lemma 2 can be applied to P_2^u with $\gamma_c = \gamma_{SD}^{\text{norm}}$, $\gamma_c \sim \text{Gamma}(1,\bar{\gamma})$, $\eta_c = \Delta_{SD}\frac{\sigma_{SD}^2}{N_S}$, $\gamma_e = \sum_{j\in J_1}\gamma_{R_jD}^{\text{norm}} + \sum_{j\in J_2}\gamma_{SR_j}^{\text{norm}}$, $\gamma_e \sim \text{Gamma}(N_R,\bar{\gamma})$, and

$$\eta_e = 2\beta\max\left\{\max_{j\in J_1}\left[\left(\frac{d^{\min}}{2}\right)^2\sigma_{R_jD}^2\right], \max_{j\in J_2}\left[\frac{\gamma_{R_jD}}{\bar{\gamma}_{R_jD}}\frac{\sigma_{SR_j}^2}{N_S}\right]\right\}$$

$$\eta_e' = \min_{j\in J_2}\left(\frac{\bar{\gamma}_{R_jD}}{\gamma_{R_jD}}\right)\min\left\{\min_{j\in J_1}\left[\left(\frac{d^{\min}}{2}\right)^2\frac{\sigma_{R_jD}^2}{N_R}\right], \min_{j\in J_2}\left[\frac{\gamma_{R_jD}}{\bar{\gamma}_{R_jD}}\frac{\Delta_{SR_j}}{N_R}\frac{\sigma_{SR_j}^2}{N_S}\right]\right\}.$$

We have thus proved that $E[P_2^u]$ also exhibits diversity order N_R+1, and hence full diversity N_R+1 is guaranteed for the average SEP at D as well as for the average SEP at each source.

Recapitulating the results pertaining to RS, we have established that:

Proposition 3: *The $(N_S,N_R,1)$ wireless cooperative network with CFNC and RS achieves throughput 1/2 sym/S/CU and full diversity with ML multiuser detection for each source and any triplet (σ_{SR}^2,σ_{SD}^2,σ_{RD}^2).*

IV. Information Exchanging Sources

In the cooperative networks discussed so far N_S sources transmit to a common destination D. As we will see in this section, sources in these networks can also exchange information among themselves. This information exchange (IE) setup has also been considered in [7], [21], and requires naturally each source to have available CSI of the $R \to S$ links. Two IE setups based on PR and RS schemes will be described in the ensuing subsections.

A. Information Exchange with Parallel Relays

Consider information exchange among sources in an $(N_S,N_R,1)$ setup with PR. The first N_R channel uses coincide with those in Subsection III-A. During CU (N_R+j), source S_i receives

$$y_{R_jS_i} = \sqrt{\alpha_j}h_{R_jS_i}\boldsymbol{\theta}_R^T\hat{\mathbf{x}}_j + n_{R_jS_i} \quad (31)$$

where the notation is directly analogous to (22) after replacing D with S_i, $i = 1,\dots,N_S$. Over $2N_R$ channel uses, each source receives N_R copies of all sources' information. Hence, the symbol rate is again 1/2 sym/S/CU and full diversity N_R is achieved here. ML demodulation per S_i yields

$$\hat{\mathbf{x}}_{S_i} = \arg\min_{\mathbf{x}'}\sum_{j=1}^{N_R}||y_{R_jS_i} - \sqrt{\alpha_j}h_{R_jS_i}\boldsymbol{\theta}_R^T\mathbf{x}'||^2$$
$$= \arg\min_{\mathbf{x}'}||(\sum_{j=1}^{N_R}\sqrt{\alpha_j}h_{R_jS_i}^*y_{R_jS_i}) - (\sum_{j=1}^{N_R}\alpha_j|h_{R_jS_i}|^2)\boldsymbol{\theta}_R^T\mathbf{x}'||^2.$$

Recall that sources know each other's symbol-combining coefficients which allows each of them to cancel its own signal from $\hat{\mathbf{x}}_{S_i}$ and figure out the other sources' information. Also, S_i only needs the product $\sqrt{\alpha}h_{R_jS_i}$, and D only needs the products $\sqrt{\alpha}h_{R_jD}$, $j = 1,\dots,N_R$.

[3]A similar relay selection scheme based on traditional decode-and-forward protocol has been proposed in [3].

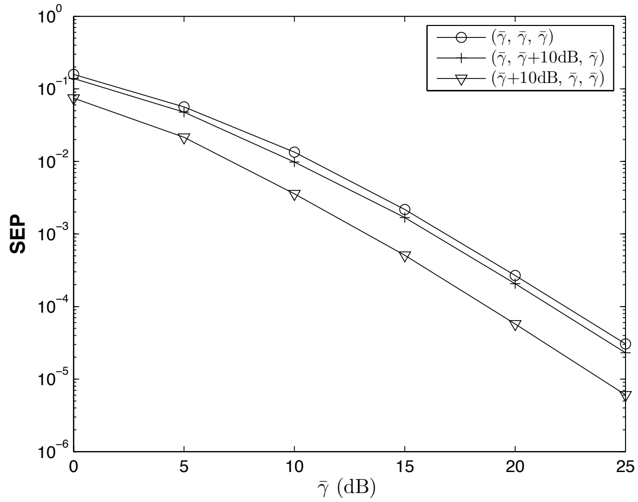

Fig. 5. SEP comparison for various channel SNR settings in a $(2, 1, 1)$ CFNC-based network.

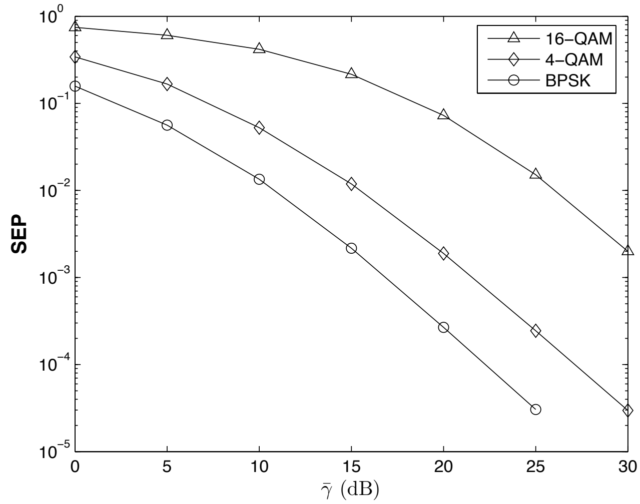

Fig. 6. SEP comparison for various constellations in a $(2, 1, 1)$ CFNC-based network.

Following similar steps as before, the SEP can be upper bounded as

$$P^v_{S_i} \leq \sum_{\varepsilon=0}^{N_R} \sum_{j=1}^{\binom{N_R}{\varepsilon}} \left\{ (N_R M - 1)^{\varepsilon+1} \exp\left(-\sum_{k=1}^{\varepsilon} \Delta_{SR_{E_k^j}} \gamma^{\min}_{SR_{E_k^j}} \right) \right.$$
$$\left. Q\left[\frac{\sqrt{2}[\sum_{l=1}^{N_R-\varepsilon} \alpha_{C_l^j} \gamma_{R_{C_l^j} S_i} - 2\beta \sum_{k=1}^{\varepsilon} \alpha_{E_k^j} \gamma_{R_{E_k^j} S_i}]}{\sqrt{\sum_{l=1}^{N_R-\varepsilon} \alpha_{C_l^j} \gamma_{R_{C_l^j} S_i} + 2\beta \sum_{k=1}^{\varepsilon} \alpha_{E_k^j} \gamma_{R_{E_k^j} S_i}}} \right] \right\} \quad (32)$$

where corresponding terms are analogous to those in (25) after replacing D with S_i, $i = 1, \ldots, N_S$. Arguing as in Subsection III-B, one can easily verify that Lemma 2 applies to (32) with $n_c = N_R - \varepsilon$ and $n_e = \varepsilon$ for each given ε. Therefore, full diversity N_R is achieved with throughput $1/2$ sym/S/CU.

B. Information Exchange with Relay Selection

It is also natural to consider sources exchanging information in an $(N_S, N_R, 1)$ setup with RS. The transmission period here includes two channel uses, where the first one is exactly the same as in the $(N, 1, 1)$ setup using a winner relay R_w. During the second channel use, S_i receives

$$y_{R_w S_i} = h_{R_w S_i} \sqrt{\alpha_w} \boldsymbol{\theta}^T \hat{\mathbf{x}}_{R_w} + n_{R_w S_i} \quad (33)$$

where corresponding terms are as in (7) except that R is replaced by R_w and D by S_i for $i = 1, \ldots, N_S$. So each source receives other sources' information from the winner relay R_w at rate $1/2$ sym/S/CU. ML demodulation at S_i yields

$$\hat{\mathbf{x}}_{S_i} = \arg\min_{\mathbf{x} \in \mathcal{A}_{\mathbf{x}}} ||y_{R_w S_i} - h_{R_w S_i} \sqrt{\alpha_w} \boldsymbol{\theta}^T \mathbf{x}||^2$$
$$= \underset{\mathbf{x}}{\operatorname{argmin}} ||(\sqrt{\alpha_w} h^*_{R_w S_i} y_{R_w S_i}) - (\alpha_w |h_{R_w S_i}|^2) \boldsymbol{\theta}^T \mathbf{x}||^2.$$

Then each source cancels its own signal from $\hat{\mathbf{x}}_{S_i}$ to figure out the other sources' information.

SEP analysis follows steps analogous to those in Subsection III-C. The main difference is that Lemma 2 no longer applies to $E[P^u]$. As will be confirmed later with simulations, IE based on RS loses diversity because although the winner relay has maximum $\alpha_w \gamma_{R_w D}$, any $\gamma_{R_w S_i}$ may be small enough to prevent collecting the full diversity at S_i.

V. Simulations

Using multiuser detection to implement ML demodulation, simulations are described in this section to compare SEPs of CFNC in various scenarios for practical SNR values. Unless specified otherwise, binary phase shift keying (BPSK) is used throughout.

With reference to Fig. 5, we consider representative magnitudes of fading scenarios where R is located either close to the sources, close to the destination, or equi-distant from both; the corresponding average SNRs ($\bar{\gamma}_{SR}$, $\bar{\gamma}_{RD}$, $\bar{\gamma}_{SD}$) in logarithmic-scale are ($\bar{\gamma}$+10dB, $\bar{\gamma}$, $\bar{\gamma}$), ($\bar{\gamma}$, $\bar{\gamma}$+10dB, $\bar{\gamma}$) and ($\bar{\gamma}$, $\bar{\gamma}$, $\bar{\gamma}$), respectively. In each case, the average SEP with CFNC-based cooperation is simulated and full diversity is verified for $\bar{\gamma}$ large enough. When $\bar{\gamma}_{SR}$ is 10dB higher than others, performance improves by approximately 3dB from the symmetric scenario. When $\bar{\gamma}_{RD}$ is 10dB higher than others, SEP improvement is not that evident, but in this case R saves transmit power as its average scaling coefficient α is smaller. Nevertheless, full diversity is achieved regardless of the power constraints and channel gains for different sources and relays. For the subsequent simulations, we fix the channel setting to ($\bar{\gamma}_{SR}$, $\bar{\gamma}_{RD}$, $\bar{\gamma}_{SD}$)=($\bar{\gamma}$, $\bar{\gamma}$, $\bar{\gamma}$).

To check modulations other than BPSK, we test 4-QAM and 16-QAM with CFNC. As shown in Fig. 6, the average SEP exhibits decreasing coding gain as the constellation size increases, while full diversity is always achieved for sufficiently high SNR. It is certainly possible to improve the coding gain by either optimizing $\boldsymbol{\theta}^T$ as in [20], or, by invoking error control coding (ECC) at the expense of rate loss.

In Fig. 7, we test a CFNC-based $(N_S, 1, 1)$ setup to validate the full-diversity claims. Since CFNC defines a one-to-one mapping from a set of multiple constellations to a larger constellation, full-diversity can be achieved for any N_S. This same mapping entails loss in coding gain as N_S increases

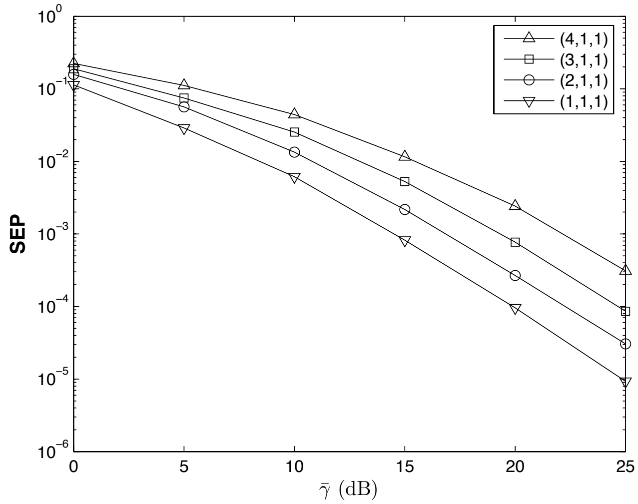

Fig. 7. SEP comparison in $(N, 1, 1)$ CFNC-based networks.

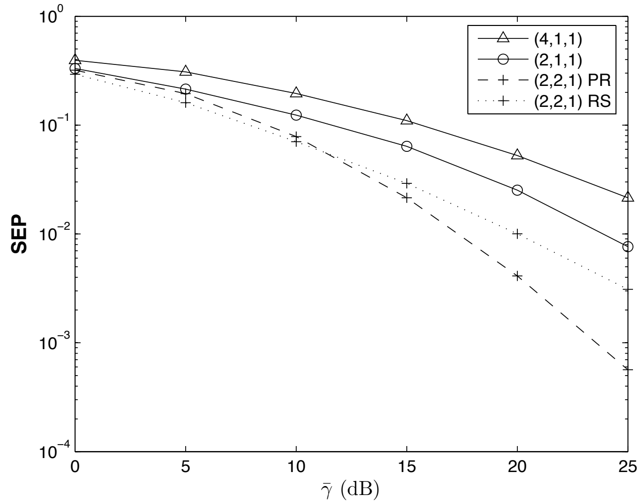

Fig. 9. SEP comparison for IE in $(N_S, N_R, 1)$ CFNC-based networks.

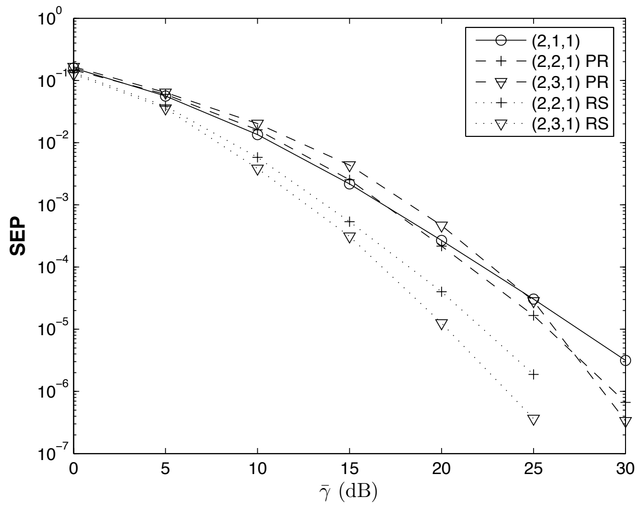

Fig. 8. SEP comparison in $(2, N_R, 1)$ CFNC-based networks.

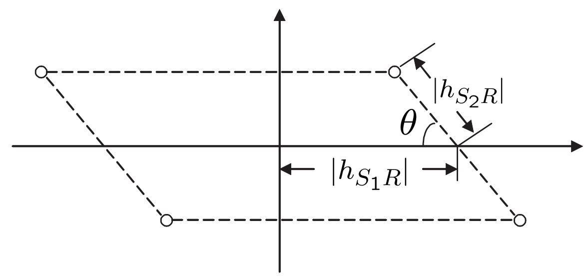

Fig. 10. The constellation received at R in a CFNC-based $(2, 1, 1)$ network with BPSK.

because the $d^{\min}$ among received symbols decreases with increasing N_S. Notice that when $N_S = 1$, the SEP coincides with that of the traditional relay network [13], as in Fig. 1(a).

For a more general $(N_S, N_R, 1)$ setup, Fig. 8 depicts the SEP with CFNC for $N_S = 2$ and increasing N_R. Both PR and RS alternatives are considered with the $(2, 1, 1)$ paradigm benchmarking all other cases. When N_R grows large with PR, a tradeoff emerges. The diversity gain grows as N_R increases while the coding gain becomes smaller because $d^{\min}$ decreases. In RS however, both diversity and coding gains increase at the same time as N_R grows large. On the other hand, RS requires D to feed back the index of the winner relay at the beginning of every frame, while PR only requires readily available local CSI.

Consider now the setup in Section IV, where sources exchange information at the same time they transmit to a common destination. Recall that the maximum achievable diversity in IE is N_R. As Fig. 9 confirms, diversity $N_R = 1$ is achieved for both $(4, 1, 1)$ and $(2, 1, 1)$ networks when SNR is large enough. For a $(2, 2, 1)$ setup, full diversity $N_R = 2$ is achieved with PR but lower diversity is effected with RS, as asserted in Section IV. From Figs. 8 and 9, the tradeoffs between PR and RS options are evident. Notice also that in Fig. 9, the coding gains are suboptimum since the relays adapt their transmit-power to the $R \to D$ rather than the $R \to S_i$ links.

VI. Conclusions

We have introduced a novel complex field network coding (CFNC) approach with attractive rate and diversity features useful for wireless cooperative networks involving multiple sources and relays. The throughput-diversity benefits of CFNC-based networks are possible when multiuser detection is employed for ML demodulation regardless of the SNR and the constellations used. The framework is general enough to allow not only for transmissions from multiple sources to a common destination but also for information exchange among sources. Parallel relay and relay selection options have been considered and found to exhibit complementary strengths.[4]

[4]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.

APPENDIX

A. Proof of Lemma 1

Let us first consider the $S_i \to R$ links in a simple $(2,1,1)$ setup with CFNC applied to BPSK symbols. Without loss of generality (w.l.o.g.), assume that $|h_{S_1R}| \geq |h_{S_2R}|$ and the phase of h_{S_1R} is 0, since R can always cancel the phase of h_{S_1R}. The constellation received at R is depicted in Fig. 10, where θ depends on the phase of h_{S_2R} ($\theta_{h_{S_2R}}$) only, and is chosen as $\theta = \pi - (\theta_{h_{S_2R}} + 3\pi/4)$ [8]. Since $\theta_{h_{S_2R}}$ is uniformly distributed over $[0, 2\pi)$, so is θ.

In this case, one can easily verify that $d_{SR}^{\min}/2 \geq \min(|h_{S_1R}||\sin\theta|, |h_{S_2R}|)$ and

$$\gamma_{SR} = (\frac{d_{SR}^{\min}}{d^{\max}})^2 (\frac{d^{\min}}{2})^2 \bar{\gamma} \geq \min(|h_{S_1R}|^2 \sin^2\theta, |h_{S_2R}|^2) \frac{\bar{\gamma}}{\beta^2}$$

where $\beta = d^{\max}/d^{\min}$ is fixed during the transmission. Since $\gamma_{SR}^{\min} = \min(|h_{S_1R}|^2, |h_{S_2R}|^2)\bar{\gamma}$, it follows that $\frac{\gamma_{SR}}{\gamma_{SR}^{\min}} \geq \frac{\sin^2\theta}{\beta^2} =: \Delta_{SR}$. Note here that Δ_{SR} is a random variable dependent on θ but independent of $\gamma_{SR}^{\min}$. Moreover, $\Delta_{SR} \geq 0$ and the equality holds if and only if $\theta = 0$ or π. Since θ is continuously distributed over $[0, 2\pi)$, we have that $\Pr(\theta = 0) = \Pr(\theta = \pi) = 0$, and hence $\Pr(\Delta_{SR} > 0) = 1$.

For a general $(N_S, N_R, 1)$ network where the physical layer relies on an M-ary constellation, if only N_S, N_R and M are finite, we consider the random variable $\Delta_{SR} \leq \frac{\gamma_{SR}}{\gamma_{SR}^{\min}}$, which is independent of $\gamma_{SR}^{\min}$ and satisfies $\Pr(\Delta_{SR} = 0) = \sum_{i=1}^{n}\sum_{j=1}^{m_i} \Pr(\Omega_i = \omega_j) = 0$, where Ω_i's are continuously distributed random variables and n, m_i are finite numbers. Again, we find $\Pr(\Delta_{SR} > 0) = 1$, and the proof for the $S \to D$ link follows similar steps as before, after replacing the h_{S_iR}'s with h_{S_iD}'s.

B. Proof of Lemma 2

Averaging P_e over γ_c and γ_e, we obtain $P_e(\eta_c, \eta_e, \eta_e')$, which is a function of η_c, η_e and η_e'. Next, we upper bound $P_e(\eta_c, \eta_e, \eta_e')$ as

$$P_e(\eta_c, \eta_e, \eta_e') \leq P_A(\eta_c, \eta_e, \eta_e') + P_B(\eta_c, \eta_e, \eta_e') \quad (34)$$

where

$$P_A(\eta_c, \eta_e, \eta_e') := \int_0^\infty \int_{\eta_e\gamma_e/\eta_c}^\infty \exp(-\eta_e'\gamma_e) \exp\left[-\frac{(\eta_c\gamma_c - \eta_e\gamma_e)^2}{\eta_c\gamma_c + \eta_e\gamma_e}\right] p(\gamma_c)p(\gamma_e)d\gamma_c d\gamma_e$$

$$P_B(\eta_c, \eta_e, \eta_e') := \int_0^\infty \int_{\eta_c\gamma_c/\eta_e}^\infty \exp(-\eta_e'\gamma_e)p(\gamma_e)p(\gamma_c)d\gamma_e d\gamma_c.$$

Let us rewrite $P_A(\eta_c, \eta_e, \eta_e')$ as

$$P_A(\eta_c, \eta_e, \eta_e') = \int_0^\infty P_A'(\eta_c, \eta_e, \gamma_e) \exp(-\eta_e'\gamma_e)p(\gamma_e)d\gamma_e \quad (35)$$

where

$$\begin{aligned} P_A'(\eta_c, \eta_e, \gamma_e) &= \int_{\eta_e\gamma_e/\eta_c}^\infty \exp\left[-\frac{(\eta_c\gamma_c - \eta_e\gamma_e)^2}{\eta_c\gamma_c + \eta_e\gamma_e}\right] p(\gamma_c)d\gamma_c \\ &\leq \int_{\eta_e\gamma_e/\eta_c}^\infty \exp\left[-(\eta_c\gamma_c + \eta_e\gamma_e - 2\sqrt{\eta_c\gamma_c\eta_e\gamma_e})\right] \\ &\quad \frac{(\gamma_c)^{n_c-1}}{(n_c-1)!\bar{\gamma}^{n_c}} \exp(-\frac{\gamma_c}{\bar{\gamma}})d\gamma_c \\ &\leq \sum_{k=0}^{2n_c-1} \left\{ \binom{2n_c-1}{k} \frac{(1+\eta_c\bar{\gamma})^{(k/2-n_c)}}{(n_c-1)!} \right. \\ &\quad \left. \exp(-\frac{\gamma_e\eta_e}{1+\eta_c\bar{\gamma}}) \left(\frac{\sqrt{\eta_c\bar{\gamma}\gamma_e\eta_e}}{1+\eta_c\bar{\gamma}}\right)^k \Gamma[n_c - \frac{k}{2}] \right\}. \end{aligned}$$

Then we can upper bound $P_A(\eta_c, \eta_e, \eta_e')$ by

$$\begin{aligned} P_A(\eta_c, \eta_e, \eta_e') &\leq \sum_{k=0}^{2n_c-1} \left\{ \binom{2n_c-1}{k} \frac{\Gamma[n_c - \frac{k}{2}]\Gamma[n_e + \frac{k}{2}]}{(n_c-1)!\Gamma[n_e]} \right. \\ &\quad \left. \left(\frac{\eta_e'}{\eta_e}\right)^{-k/2} {\eta_e'}^{-n_e} \eta_c^{-n_c} \bar{\gamma}^{-n_c-n_e} \right\} \\ &= \kappa_A(\eta_c, \eta_e, \eta_e')\bar{\gamma}^{-n_c-n_e}. \end{aligned} \quad (36)$$

For $P_B(\eta_c, \eta_e, \eta_e')$, integration with respect to γ_c yields

$$\begin{aligned} &P_B(\eta_c, \eta_e, \eta_e') \\ &= \int_0^\infty \int_{\eta_c\gamma_c/\eta_e}^\infty \frac{\exp(-\eta_e'\gamma_e)(\gamma_e)^{n_e-1}}{(n_e-1)!\bar{\gamma}^{n_e}} \exp(-\frac{\gamma_e}{\bar{\gamma}})d\gamma_e p(\gamma_c)d\gamma_c \\ &= (1+\eta_e'\bar{\gamma})^{-n_e} \sum_{k=0}^{n_e-1} \left\{ \frac{1}{k!}(\eta_e' + \frac{1}{\bar{\gamma}})^k \frac{\Gamma[n_c+k]}{(n_c-1)!} \right. \\ &\quad \left. (\frac{\eta_c}{\eta_e})^k \bar{\gamma}^{-n_c} (\frac{\eta_c + \eta_e + \eta_e'\eta_c\bar{\gamma}}{\eta_e\bar{\gamma}})^{-n_c-k} \right\} \\ &\leq \bar{\gamma}^{-n_c-n_e} \sum_{k=0}^{n_e-1} \frac{\Gamma[n_c+k]}{k!(n_c-1)!} (\eta_e')^{-n_e-n_c} (\frac{\eta_c}{\eta_e})^{-n_c} \\ &= \kappa_B(\eta_c, \eta_e, \eta_e')\bar{\gamma}^{-n_c-n_e}. \end{aligned} \quad (37)$$

From (36) and (37), we have

$$P_e(\eta_c, \eta_e, \eta_e') \leq \kappa(\eta_c, \eta_e, \eta_e')\bar{\gamma}^{-n_c-n_e} \quad (38)$$

where $\kappa(\eta_c, \eta_e, \eta_e') = \kappa_A(\eta_c, \eta_e, \eta_e') + \kappa_B(\eta_c, \eta_e, \eta_e')$ is a random variable satisfying $\Pr(\kappa(\eta_c, \eta_e, \eta_e') > 0) = 1$; and hence, $E[P_e(\eta_c, \eta_e, \eta_e')] \leq E[\kappa(\eta_c, \eta_e, \eta_e')]\bar{\gamma}^{-n_c-n_e}$, where $E[\kappa(\eta_c, \eta_e, \eta_e')] > 0$ is a constant not dependent on $\bar{\gamma}$. Thus, when $\bar{\gamma} \to \infty$, $E[P_e(\eta_c, \eta_e, \eta_e')]$ achieves diversity order $N := n_c + n_e$.

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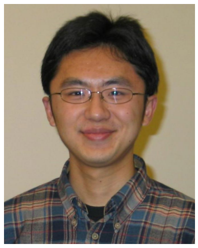

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