On the Probability of Error for Multichannel Reception of Binary Signals

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ABSTRACT: In this paper a derivation of the probability of error which arises in 1) adaptive multichannel reception of binary signals and 2) multichannel communication with binary signaling over channels that are characterized by both a specular (nonfading or constant) component and a Rayleigh fading component is presented.

Introduction

Several years ago, Price[1] published a paper in which he described the operation and gave the performance of an adaptive receiver that is used with binary signaling in a multichannel communication system. The performance of the receiver was given in terms of the probability of error that Price obtained by evaluating a double integral.[1],[2] Price also showed that the double integral is related to the probability distribution function of a doubly noncentral F-distribution.[3]

Since the publications of Price's works, two other authors, Hingorani[4],[5] and Bello,[6] have shown that the expression for the probability of error for a multichannel transmitter–receiver system that employs binary signaling in communicating over channels characterized by both a specular (nonfading or constant) component and a Rayleigh fading random component can be expressed in the form of the double integral evaluated by Price. The underlying reason for the similarity in the expressions for the error probability for these two seemingly different receivers is that the decision variable, in both cases, is given in general by the quadratic form[1]

$$D = \sum_{k=1}^{L} [A|X_k|^2 + B|Y_k|^2 + CX_kY_k^* + C^*X_k^*Y_k] \quad (1)$$

in complex-valued Gaussian random variables. A, B, and C are constants; X_k and Y_k are a pair of correlated complex-valued Gaussian random variables. For the channels considered in these works, the L pairs $\{X_k, Y_k\}$ are mutually statistically independent and identically distributed.

The probability of error is the probability that D is less than zero. This probability can be expressed in the form of the double integral evaluated by Price.[1]–[3] The purpose of this paper is to present an alternative derivation of this error probability.

[1] Complex conjugation is denoted by an asterisk.

Probability of Error for the General Quadratic Form in Gaussian Variables

The computation begins with the characteristic function, denoted by $\phi_D(iv)$, of the general quadratic form that has been given by Turin.[7] The probability that D is less than zero, denoted here as the probability of error $P(e)$, is

$$P(e) = \Pr[D < 0] = \int_{-\infty}^{0} p(D)dD \quad (2)$$

where $p(D)$, the probability density function of D, is related to $\phi_D(iv)$ by the Fourier transform, i.e.,

$$p(D) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_D(iv) \exp[-ivD]dv.$$

Hence

$$P(e) = \int_{-\infty}^{0} dD \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_D(iv) \exp[-ivD]dv. \quad (3)$$

Let us interchange the order of integrations and carry out first the integration with respect to D. The result is

$$P(e) = \frac{-1}{2\pi i}\int_{-\infty+i\epsilon}^{\infty+i\epsilon} \frac{\phi_D(iv)}{v}\, dv \quad (4)$$

where a small positive number ϵ has been inserted in order to move the path of integration away from the singularity at $v = 0$, and which must be positive in order to allow for the interchange in the order of integrations.

Since D is the sum of statistically independent random variables, the characteristic function of D factors into a product of L characteristic functions, with each function corresponding to the individual random variables d_k, where

$$d_k = A|X_k|^2 + B|Y_k|^2 + CX_kY_k^* + C^*X_k^*Y_k.$$

The characteristic function of d_k is[4]–[6]

$$\phi_{d_k}(iv) = \frac{v_1v_2}{(v + iv_1)(v - iv_2)} \times \exp\left\{\frac{v_1v_2(-v^2\alpha_{1k} + iv\alpha_{2k})}{(v + iv_1)(v - iv_2)}\right\} \quad (5)$$