Coupling Matrix Synthesis for a New Class of Microwave Filter Configuration

R. J. Cameron[(1)], J. C. Faugere[(2)] and F. Seyfert[(3)]

[(1)] Com Dev Space (UK), [(2)]Univ. Paris VI (France), [(3)]INRIA (France)

Abstract **— In this paper a new approach to the synthesis of coupling matrices for microwave filters is presented. The new approach represents an advance on existing direct and optimization methods for coupling matrix synthesis in that it will exhaustively discover all possible coupling matrix solutions for a network if more than one exists. This enables a selection to be made of the set of coupling values, resonator frequency offsets, parasitic coupling tolerance etc that will be best suited to the technology it is intended to realize the microwave filter with. To demonstrate the use of the method, the case of the recently – introduced 'extended box' (EB) coupling matrix configuration is taken. The EB represents a new class of filter configuration featuring a number of important advantages, one of which is the existence of multiple coupling matrix solutions for each prototype filtering function, eg 16 for 8th degree cases. This case is taken as an example to demonstrate the use of the synthesis method – yielding one solution suitable for dual-mode realization and one where some couplings are small enough to neglect.**

Index Terms **— Coupling matrix, filter synthesis, Groebner basis, inverted characteristic, multiple solutions.**

I. Introduction

In reference [1], a synthesis method for the 'Box Section' configuration for microwave filters was introduced. Box sections are able to realize a single transmission zero each, and have an important advantage that no 'diagonal' inter-resonator couplings are required to realize the asymmetric zero, as would the equivalent trisection. Also the frequency characteristics are reversible by retuning the resonators alone, retaining the same values and topology of the inter-resonator couplings.

The first feature leads to particularly simple coupling topologies, and is suitable for realization in the very compact waveguide or dielectric dual-mode resonator cavity, whilst the ability to reverse the characteristics by retuning makes the box-filter useful for diplexer applications, the same structure being usable for the complementary characteristics of the two channel filters.

Reference [1] continued on to introduce the extended box configuration for filter degrees $N > 4$, able to realize a maximum of $(N–2)/2$ (N even) or $(N–3)/2$ (N odd) symmetric or asymmetric transmission zeros. Fig. 1 gives EB networks of even degree 4 (basic box section), 6, 8 and 10, showing the particularly simple ladder network form of the EB configuration. In each case, the input and output are from opposite corners of the ladder network. The EB network also retains the property of giving lateral inversion of the frequency characteristics by retuning of the resonators alone.

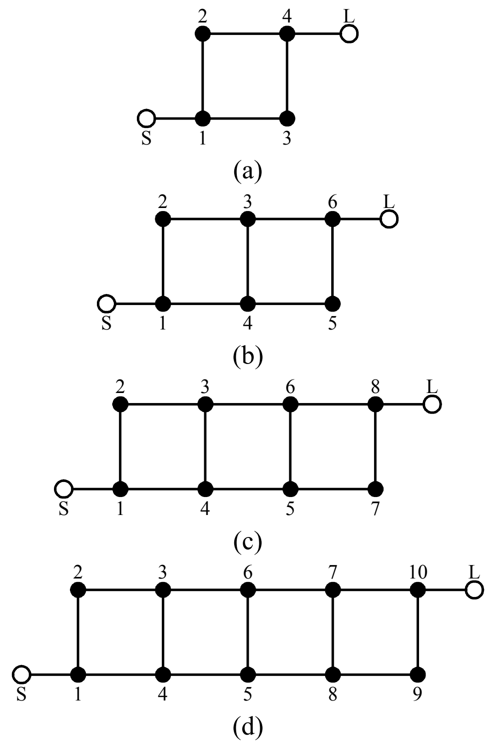

Fig. 1. Coupling and routing diagrams for extended box section networks: (a) 4th degree (basic box section) (b) 6th degree (c) 8th degree (d) 10th degree.

The prototype coupling matrix for the EB network may be easily synthesized in the folded or 'arrow' forms. However it appears that there is no simple closed form equation or procedure that may be used to transform the folded or arrow coupling matrix to the EB form. In [1] a method was described which is essentially the reverse of the general sequence that reduces any coupling matrix to the folded form, for which a regular sequence of rotation pivots and angles does exist. Using this method means that some of the rotation angles cannot be determined by calculation from the pre-transform coupling matrix (as can be done from the 'forward method') and so they have to be determined by optimization. Other methods (eg. [2]) are also known to produce a solution.

Although most target CM configurations (eg propagating in-line) have one or two unique solutions, the EB configuration is distinct in having multiple solutions, all returning exactly the same performance characteristics under analysis as the original prototype folded or arrow configuration. The solutions converged upon by existing optimization methods

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tend to be dependent upon the starting values given to the coupling values or rotation angles, and it can never be guaranteed that all possible solutions have been found. This paper describes a new method making use of computer algebra techniques that exhaustively discovers all the solutions for the given CM topology for the coupling elements, including those with complex values (which of course are discarded from the solutions considered for the realization of the hardware).

Having a range of solutions enables a choice to be made of the coupling value set most suited to the technology it is intended to realize the filter with. Considerations influencing the choice include ease of the design of the coupling elements, minimization of parasitic couplings or resonator frequency offsets. Some of the CM solutions may contain coupling elements with values small enough to be ignored without damage to the overall electrical performance of the filter, so simplifying the manufacture and tuning processes.

In the following section a general proof will be given for the inversion of the frequency characteristics of a network. This is followed by a description of the multi-solution synthesis method, applicable to the EB network and others that support multiple solutions. Finally an example is taken of an asymmetric 8th degree characteristic with 3 transmission zeros, suitable for realization in dual-mode waveguide or dielectric resonator cavities.

II. Reversed Frequency Characteristics

We say that a matrix M is "odd" (resp. "even") if the following holds: for all indices (i,j) such that $(i+j)$ is even (resp. "odd") we have $M[i,j]=0$. It is straightforward that every matrix M decomposes uniquely in the sum of its odd part (denoted M_o) and even part (M_e). Now if M is the (NxN) coupling matrix of a lossless filter we denote by $y_{i,j}[M]$ and $S_{ij}[M]$ the corresponding reduced admittance and scattering parameters (the input and output loads are fixed). The following properties can be used to reverse in a simple manner the frequency properties of a filter.

- $y_{11}[M_o - M_e](s) = -y_{11}[M_o + M_e](-s)$ and the same is true for y_{22}
- $y_{12}[M_o - M_e](s) = (-1)^N y_{12}[M_o + M_e](-s)$

On the imaginary axis $s = j\omega$,

- $S_{11}[M_o - M_e](j\omega) = (S_{11}[M_o + M_e](-j\omega))^*$ and the same is true for S_{22}.
- $S_{12}[M_o - M_e](j\omega) = (-1)^{N+1} (S_{12}[M_o + M_e](-j\omega))^*$

Proof: From the fact that the product of two square matrices with same parity is "even" and the product of two square matrices with different parities is "odd" one proves by induction on k that,

$$\mathrm{Odd}((M_o - M_e)^k = (-1)^{k+1}\mathrm{Odd}((M_o + M_e)^k) \qquad (1)$$

$$\mathrm{Even}((M_o - M_e)^k) = (-1)^k\mathrm{Even}((M_o + M_e)^k) \qquad (2)$$

where Odd( ) and Even( ) means respectively taking the odd and the even parts. Now recalling that:

$$Y(s) = C(sI - jM)^{-1}C^t = \sum_{k=0}^{\infty} \frac{C\, j^k M^k C^t}{s^{k+1}} \qquad (3)$$

with:

$$C = \begin{bmatrix} \sqrt{R_1} & \ldots 0 \ldots & 0 \\ 0 & \ldots 0 \ldots & \sqrt{R_N} \end{bmatrix}$$

(R_1, R_N input/output termination impedances), and plugging in the relations (1-2) yields directly the formulae for Y. Finally the classical formula $S = (I - Y)/(I + Y)$ and the fact that Y is pure imaginary on the imaginary axis lead to the formulae for S.

In effect this means that to reverse the frequency characteristic of any coupling matrix, elements with indices (i, j) where $(i+j)$ = even are changed in sign, whilst those where $(i+j)$ = odd retain their original sign. Thus for a ladder network such as the EB network, the elements on the principal diagonal, each of whose indices add to an even integer, need to be changed in sign (ie. conjugate - tuned), to laterally invert the network's response with frequency. All off-diagonal elements retain their original sign, except for 4th, 8th, 12th… degree cases where the indices of the last two couplings (eg M_{68} and M_{57} in the 8th degree case, see Fig. 1c) have an index sum that is even. However since they always occur in pairs, they too may retain their original sign.

III. A General Framework for the Coupling Matrix Synthesis Problem

In this section we work with a fixed coupling topology, that is we are given a set of independent non-zero couplings associated to a low pass prototype of some filter with N resonators. Starting with numerical values for the couplings and the i/o loads one can easily compute the admittance matrix using equation (3). The coupling matrix synthesis problem is actually about inverting the latter procedure: given an admittance matrix we want to find values for the input/output loads and couplings that realize it. In order to formalize this we give a name to the mapping that builds the admittance matrix from the free electrical parameters and we define:

$$T{:}p = (\sqrt{R_1}, \sqrt{R_N} \ldots M_{i,j}) \rightarrow (CC^t, \ldots CM^k C^t, \ldots CM^{2N-1}C^t)$$

The above definition is justified by the fact that the admittance matrix is entirely determined by the first $2N$ coefficients of its power expansion at infinity [3].

Now suppose that each of the electrical parameters move around in the complex plane: what about the corresponding set of admittance matrices? The latter can be identified with the image by T of C^r (C is here the field of complex numbers) where r is the number of free electrical parameters. We call this set V (=T(C^r)) and refer to it as the set of admissible admittance matrices with respect to the coupling topology.

In this setting the coupling matrix synthesis problem is the following: given an element w in V compute the solution set of:

$$T(p) = w \qquad (4)$$

Now from the definition of T it follows that equation (4) is a non-linear polynomial system with *r* unknowns, namely: the square roots of the i/o loads and the free couplings of the topology. From the polynomial structure of the latter system we can deduce following mathematical properties (we will take them here for granted):

- Equation 4 has a finite number of solutions for all generic w in V if and only if the differential of T is generically of rank *r*. In this case we will say that the coupling topology is non-redundant.
- The number of complex solutions of the equation 4 is generically constant with regard to w in V. Because of the sign symmetries this number is a multiple of 2^N and can therefore be written as $m2^N$. The number *m* is the number of complex solutions up to sign symmetries and we will call it the "reduced order" of the coupling geometry.
- Note: The non-redundancy property ensures that a coupling geometry is not over-parameterized which would yield a continuum of solutions to our synthesis problem. Fig. 2 illustrates this with a 6th degree topology:
- if no diagonal couplings are present (as suggested by the grey dots in Fig. 2), the topology is redundant, i.e the synthesis problem admits an infinite number of solutions.
- If, for example, the coupling (1,4) is removed, the topology becomes non-redundant and is adapted to a 6-2 symmetric filtering characteristic.
- Finally, if diagonal couplings are allowed, the topology becomes non-redundant, and is actually the 6th degree extended box topology adapted to a 6-2 asymmetric filtering characteristic.

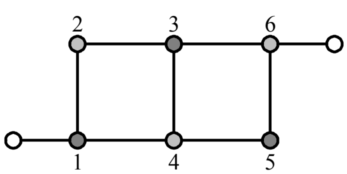

Fig. 2. Redundant topology.

In the next section we briefly explain how multivariate polynomial systems can be solved by means of Groebner basis computations.

A. Groebner Basis

As an example of the use of Groebner basis, suppose we are given the following system:

$$\begin{cases} x^2 + 2xy + 1 = 0 & (a) \\ x^2 + 3xy + y + 2 = 0 & (b) \end{cases}$$

By combining equations we get the following polynomial consequences:

$$\begin{array}{lll} (b)-(a): & xy + y + 1 = 0 & (c) \\ (c)x-(b)y: & 3xy^2 - yx - x + y^2 + 2y = 0 & (d) \\ (d)-(c)y: & -yx - x - 2y^2 - y = 0 & (e) \\ (e)+(c): & -x - 2y^2 + 1 = 0 & (f) \\ (f)y+(c): & -2y^3 + 2y + 1 = 0 & (g) \end{array}$$

Note that equation (*g*) is a univariate polynomial in the unknown *y*. Solving the latter numerically yields the following 3-digit approximations for *y*: {–0.56+0.25*j*, –0.56–0.25*j*, 1.19} and from (6) we get the corresponding values for x = {0.42–0.61*j*, 0.42+0.61*j*, -1.84}. Now we can verify that the latter three pairs of values for (*x,y*) are also solutions of (*a*) and (*b*) and therefore the only three solutions of our original system. Equations (*f*) and (*g*) are what is called a Groebner basis (for the lex. ordering) [4] of our original system and allows to reduce the resolution of a multivariate polynomial system to the one of a polynomial in a single unknown. Actually this kind of reduction can always be done as long as the original system has only isolated solutions [5]. For our synthesis problem this is ensured by the non-redundancy of the considered coupling topology.

In practice, computing a Groebner basis can be computationally very costly and therefore the use of specialized algorithms and their effective software implementation is strongly recommended. In this work we have used the tool Fgb [6].

Table I below summarizes the reduced order and the number of real solutions observed for a particular filtering characteristic for each of the EB networks of Fig.1. Whereas the reduced order depends only on the coupling geometry, the number of real solutions depends on the prototype characteristic the network is realizing (position of transmission zeros (TZs), return loss, etc…) and is, by definition, bounded from above by the reduced order.

TABLE I
RED. ORDER & OBSERVED NUMBER OF REAL SOLUTIONS

N	Max. No. of TZs	Reduced Order	Observed No. of Real Solutions
4	1	2	2
6	2	8	6
8	3	48	16
10	4	384	58

B. 8th Degree Extended Box Filter.

As an application we will consider the synthesis of an 8th degree filter in extended box configuration (see Fig. 1c). Using a computer algebra system (eg. Maple) we check that

that this topology is non-redundant and from the application of the minimum path rule we conclude that the set of admissible admittances consists of rational reciprocal matrices of degree 8 with at most 3 transmission zeros. Using classical quasi-elliptic synthesis techniques an eighth degree filtering characteristic is designed with a 23dB return loss and three prescribed TZs producing one rejection lobe level of 40dB on the lower side and two at 40dB on the upper side (see Fig. 3a).

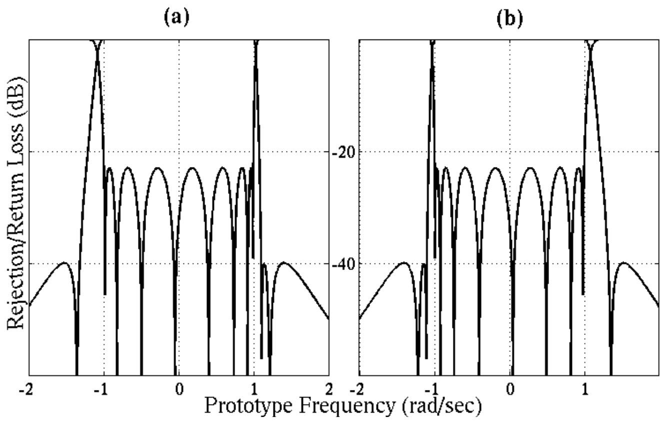

Fig. 3. (a) Original and (b) inverted rejection and return loss performance of an 8-3 asymmetric characteristic in EB configuration

Now computing the $2N$ first terms of the power expansion of the admittance matrix yields the left hand term of equation (4) which in turn is solved using Groebner basis computations and leads to following results:

- the reduced order of the topology is 48
- for this particular filtering characteristic, 16 of the 48 solutions are real valued.

Only the real solutions have a physical interpretation and are therefore of practical interest.

The criterion used to choose the best coupling matrix out of the 16 realizable ones should depend on the hardware implementation of the filter. Having in mind a realization with dual mode cavities, we choose to select solutions where the asymmetry between the two "arms" of each cross-iris is maximized in order to minimize parasitic couplings. The best ratios between couplings of the relevant pairs (M_{14}, M_{23}), (M_{36}, M_{45}) and (M_{57}, M_{68}) are found for the solution shown in Fig. 4a, where each cross-iris has one of its coupling values being at least 5 times larger than the other one.

Fig. 4b illustrates that sometimes solutions emerge which have very small values for certain couplings (M_{12} and M_{78} in this case), which may be safely omitted for the implementation without damaging the final response of the network. In this case a quasi cul-de-sac network is produced, similar to the 8-3 example given in [1].

Finally, using the result of Section II it is shown that only the resonators need to be retuned in order to obtain an inverted characteristic. Fig. 3b shows the rejection and return loss obtained from the coupling matrices of Fig. 3 when the signs of their diagonal elements $M_{i,i}$ are changed.

0.0107	-0.2904	0	-0.8119	0	0	0	0
-0.2904	-0.9804	0.1081	0	0	0	0	0
0	0.1081	0.0605	0.5475	0	0.5984	0	0
0.8119	0	0.5475	0.1384	-0.0663	0	0	0
0	0	0	-0.0663	0.0152	0.5334	0.6782	0
0	0	0.5984	0	0.5334	0.0226	0	-0.1260
0	0	0	0	0.6782	0	0.0113	0.8530
0	0	0	0	0	-0.126	0.8530	0.0107

(a)

0.0107	0.0001	0	-0.2464	0	0	0	0
0.0001	-0.9590	0.2094	0	0	0	0	0
0	0.2094	0.0498	0.4681	0	-0.4681	0	0
-0.2464	0	0.4681	0.0115	0.3744	0	0	0
0	0	0	0.3744	-0.0439	0.3744	0.8165	0
0	0	-0.4681	0	0.3744	0.0115	0	0.8623
0	0	0	0	0.8165	0	0.1975	0.0001
0	0	0	0	0	0.8623	0.0001	0.0107

(b)

Fig. 4. 'NxN' coupling matrices for an 8-3 asymmetric prototype: a) EB configuration, b) 'cul-de-sac' configuration. $R_1 = R_N = 1.0878$

CONCLUSION

In this paper, a new method for the synthesis of the full range of coupling matrices for networks that support multiple solutions is presented. An example is made of the Extended Box network, demonstrating that a choice may be made for coupling values optimal for a dual mode realization in waveguide. In addition, a knowledge of which solutions are possible is important when reconstructing the coupling matrix from measured data, during development or computer-aided tuning (CAT) processes, for example. Also the property of reversibility of frequency characteristics by tuning alone is proved.

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